

CHAPTER 1

DPP 1.1

Single Correct Answer Type

1. (c) $5^x \cdot \sqrt[3]{8^{x-1}} = 500, x \in \mathbb{N}$

$$\Rightarrow 5^x \cdot 8^{\frac{x-1}{3}} = 500$$

$$\Rightarrow 5^x \cdot 2^{\frac{3(x-1)}{3}} = 5^3 \cdot 2^2$$

$$\Rightarrow x = 3 \text{ and } \frac{3(x-1)}{3} = 2$$

$$\Rightarrow x = 3$$

2. (a) Let $\log_{175} 5x = \log_{343} 7x = k$

$$\Rightarrow 5x = 175^k \text{ and } 7x = 343^k$$

$$\Rightarrow \frac{5}{7} = \left(\frac{175}{343}\right)^k$$

$$\Rightarrow k = \frac{1}{2}$$

$$\text{Now, } 5x = (175)^{1/2}$$

$$\Rightarrow x = \sqrt{7}$$

$$\Rightarrow x^4 - 2x^2 + 7 = 42$$

$$\Rightarrow \log_{42}(x^4 - 2x^2 + 7) = 1$$

3. (a) $\log_{10} \left(\frac{\sqrt{6-2\sqrt{5}} + \sqrt{6+2\sqrt{5}}}{\sqrt{2}} \right)$

$$= \log_{10} \left(\frac{(\sqrt{5}-1) + (\sqrt{5}+1)}{\sqrt{2}} \right)$$

$$= \log_{10} \left(\frac{2\sqrt{5}}{\sqrt{2}} \right)$$

$$= \log_{10} \sqrt{10} = \frac{1}{2}$$

4. (c) $2^{\frac{4}{x}-x-3} > 2^0$

$$\Rightarrow \frac{4}{x} - x - 3 > 0$$

$$\Rightarrow \frac{4-x^2-3x}{x} > 0$$

$$\Rightarrow \frac{x^2+3x-4}{x} < 0$$

$$\Rightarrow \frac{(x+4)(x-1)}{x} < 0$$

$$\Rightarrow x \in (-\infty, -4) \cup (0, 1)$$

5. (d) Let $\log_4 A = \log_6 B = \log_9(A+B) = x$

$$\Rightarrow A = 4^x, B = 6^x \text{ and } A+B = 9^x$$

$$\Rightarrow 4^x + 6^x = 9^x$$

$$\Rightarrow 2^{2x} + 2^x \cdot 3^x = 3^{2x}$$

$$\Rightarrow (3/2)^{2x} - (3/2)^x - 1 = 0$$

$$\Rightarrow \left(\frac{3}{2}\right)^x = \frac{B}{A} = \frac{\sqrt{5}+1}{2}$$

6. (c) Let $x = \frac{1}{2\sqrt{3}} \sqrt{6 - \frac{1}{2\sqrt{3}}} \sqrt{6 - \frac{1}{2\sqrt{3}}} \sqrt{6 - \frac{1}{2\sqrt{3}}} \dots \infty$

$$\Rightarrow x = \frac{1}{2\sqrt{3}} \sqrt{6-x}$$

$$\Rightarrow 12x^2 + x - 6 = 0$$

$$\Rightarrow (3x-2)(4x+3) = 0$$

$$\Rightarrow x = \frac{2}{3}$$

$$\Rightarrow \log_9 \left(\frac{1}{2\sqrt{3}} \sqrt{6 - \frac{1}{2\sqrt{3}}} \sqrt{6 - \frac{1}{2\sqrt{3}}} \sqrt{6 - \frac{1}{2\sqrt{3}}} \dots \infty \right)$$

$$= \log_9 \left(\frac{2}{3} \right) = -\frac{1}{2}$$

7. (c) $\log_{x^2+6x+8} (\log_{2x^2+2x+3} (x^2-2x)) = 0$

$$\therefore \log_{2x^2+2x+3} (x^2-2x) = 1$$

$$\therefore x^2 - 2x = 2x^2 + 2x + 3$$

$$\Rightarrow x^2 + 4x + 3 = 0$$

$$\Rightarrow (x+1)(x+3) = 0$$

$$\therefore x = -1, -3$$

$$\text{But for } x = -3, x^2 + 6x + 8 < 0$$

$$\therefore x = -1$$

8. (a) $10^{\log_p (\log_q (\log_r x))} = 1$

$$\Rightarrow \log_p (\log_q (\log_r x)) = 0$$

$$\Rightarrow \log_q (\log_r x) = 1$$

$$\therefore \log_r x = q$$

$$\Rightarrow x = r^q$$

$$\log_q (\log_r (\log_p x)) = 0$$

$$\Rightarrow \log_r (\log_p x) = 1$$

$$\Rightarrow \log_p x = r$$

$$\Rightarrow x = p^r$$

$$\text{From (1) and (2), } r^q = p^r$$

$$\Rightarrow p = r^{q/r}$$

DPP 1.2

Single Correct Answer Type

1. (c) $\log_2 15 \times \log_{1/6} 2 \times \log_3 1/6$

$$\begin{aligned} &= \frac{\log 15}{\log 2} \cdot \frac{\log 2}{\log 1/6} \cdot \frac{\log 1/6}{\log 3} \\ &= \frac{\log 15}{\log 3} = \frac{\log 3 + \log 5}{\log 3} \\ &= 1 + \log_3 5 > 1 + 1 \end{aligned}$$

2. (c) $\log_{140} 63 = \frac{\log_7 7 + 2 \log_7 3}{2 \log_7 2 + \log_7 7 + \log_7 5}$

$$= \frac{1 + 2ac}{2c + 1 + abc}$$

3. (a) $\frac{\log_2 x}{4} = \frac{\log_2 y}{6} = \frac{\log_2 z}{3k}$

$$\begin{aligned} &= \frac{3 \log_2 x + 2 \log_2 y + \log_2 z}{12 + 12 + 3k} \\ &= \frac{\log_2 (x^3 y^2 z)}{24 + 3k} \\ &= \frac{0}{24 + 3k} \end{aligned}$$

$$\Rightarrow 24 + 3k = 0 \Rightarrow k = -8$$

4. (d) We have $\log_4 (k+4) - \log_4 x = 0.5$

$$\Rightarrow \log_4 \frac{k+4}{k} = 0.5$$

$$\Rightarrow \frac{k+4}{k} = 2$$

$$\Rightarrow k = 4$$

5. (c) $\frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3}$

$$\begin{aligned} &= (3 + \log_3 5)(1 + \log_3 5) - \log_3 5 \log_3 405 \\ &= (3 + \log_3 5)(1 + \log_3 5) - \log_3 5 \log_3 (81 \times 5) \\ &= (3 + \log_3 5)(1 + \log_3 5) - \log_3 5 (4 + \log_3 5) \\ &= 3 \end{aligned}$$

6. (c) $\frac{1}{\log_{bc} abc} + \frac{1}{\log_{ca} abc} + \frac{1}{\log_{ab} abc}$

$$\begin{aligned} &= \log_{abc} bc + \log_{abc} ca + \log_{abc} ab \\ &= \log_{abc} a^2 b^2 c^2 = 2 \end{aligned}$$

7. (a) $\log_a N \cdot \log_b N + \log_b N \cdot \log_c N + \log_c N \cdot \log_a N$

$$\begin{aligned} &= \frac{1}{\log_N a \log_N b} + \frac{1}{\log_N b \log_N c} + \frac{1}{\log_N c \log_N a} \\ &= \frac{\log_N a + \log_N b + \log_N c}{(\log_N a)(\log_N b)(\log_N c)} \\ &= \frac{\log_N abc}{\log_a N \log_b N \log_c N} \\ &= \frac{\log_a N \cdot \log_b N \cdot \log_c N}{\log_{abc} N} \end{aligned}$$

8. (b) $A \log_{200} 5 + B \log_{200} 2 = C$

$$\therefore A \log 5 + B \log 2 = C \log 200 = C \log(5^2 \cdot 2^3)$$

$$\therefore A \log 5 + B \log 2 = 2C \log 5 + 3C \log 2$$

$$\therefore A = 2C \text{ and } B = 3C$$

For no common factor greater than 1, $C = 1$

$$\therefore A = 2; B = 3$$

$$\therefore A + B + C = 6$$

9. (a) $(2^{\log_6 18}) \cdot (3^{\log_6 3})$

$$\begin{aligned} &= (2^{\log_6 6 + \log_6 3}) \cdot (3^{\log_6 3}) \\ &= 2(2^{\log_6 3}) \cdot (3^{\log_6 3}) \\ &= 2 \cdot 6^{\log_6 3} \\ &= 2 \cdot 3^{\log_6 6} \\ &= 2 \times 3 = 6 \end{aligned}$$

10. (b) $\log_3 c = 3 + \log_3 a$

$$\Rightarrow \log_3 \frac{c}{a} = 3$$

$$\Rightarrow c = 27a$$

$$\log_a b = 2 \text{ and } \log_b c = 2$$

$$\Rightarrow \log_a b \cdot \log_b c = 4$$

$$\Rightarrow \log_a c = 4$$

$$\Rightarrow c = a^4$$

From (1) and (2), we get

$$a = 3 \text{ and } c = 81$$

$$\therefore \text{from } \log_a b = 2, \text{ we get } b = a^2 = 9$$

$$\Rightarrow \frac{c}{ab} = 3$$

11. (d) $\log_{10} 154 = \log_{10} 2 + \log_{10} 7 + \log_{10} 11$

Now $\log_2 10 = p$

$$\Rightarrow \frac{1}{\log_{10} 2} = p$$

$$\Rightarrow \log_{10} 2 = \frac{1}{p}$$

$$\frac{\log_e 10}{\log_e 7} = q$$

$$\Rightarrow \frac{1}{\log_{10} 7} = q$$

S.4 Trigonometry

$$\Rightarrow \log_{10} 7 = \frac{1}{q} \quad (2)$$

$$\text{And } 11^r = 10$$

$$\Rightarrow r \log_{10} 11 = 1$$

$$\Rightarrow \log_{10} 11 = \frac{1}{r} \quad (3)$$

$$\therefore \log_{10} 154 = \frac{1}{p} + \frac{1}{q} + \frac{1}{r} = \frac{pq + qr + rp}{pqr}$$

$$12. (a) \text{ Consider } a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}$$

$$\therefore \sqrt{\log_a b} = \sqrt{\log_b a} \cdot \log_a b \text{ (taking log with base 'a')}$$

$$\Rightarrow 1 = \sqrt{\log_b a} \cdot \sqrt{\log_a b} \text{ which is true}$$

$$\therefore \text{ from } 3^{(\log_3 7)^x} = 7^{(\log_7 3)^x}, x = \frac{1}{2}$$

$$13. (c) \begin{aligned} & 4^{5 \log_4 \sqrt{2} (3 - \sqrt{6}) - 6 \log_8 (\sqrt{3} - \sqrt{2})} \\ &= 4^{2 \log_2 (\sqrt{3} (\sqrt{3} - \sqrt{2})) - 2 \log_2 (\sqrt{3} - \sqrt{2})} \\ &= 4^{2 \log_2 \sqrt{3}} \\ &= 2^{\log_2 (\sqrt{3})^4} \\ &= 9 \end{aligned}$$

$$14. (b) \begin{aligned} & \frac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_6 \sqrt{3}}}}{409} \left((\sqrt{7})^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6} \right) \\ &= \frac{9^{\log_9 5^2} + 3^{\log_3 (\sqrt{6})^3}}{409} (7^{\log_7 25} - 5^{\log_5 6^{3/2}}) \\ &= \frac{25 + 6\sqrt{6}}{409} [25 - 6\sqrt{6}] \\ &= \frac{625 - 216}{409} = \frac{409}{409} = 1 \end{aligned}$$

$$15. (b) \text{ We have } N = 6^{\log_{10} 40} \cdot 5^{\log_{10} 36}$$

$$\begin{aligned} \therefore \log_{10} N &= \log_{10} 40 \log_{10} 6 + \log_{10} 36 \log_{10} 5 \\ &= \log_{10} 6 [\log_{10} 40 + \log_{10} 25] \\ &= \log_{10} 6 [\log_{10} 1000] \\ &= \log_{10} (6)^3 \\ \therefore N &= 6^3 = 216 \end{aligned}$$

DPP 1.3

Subjective Type

$$\begin{aligned} 1. & \log_3 (\log_2 x) + \log_{1/3} (\log_{1/2} y) = 1 \\ \Rightarrow & \log_3 (\log_2 x) - \log_3 (-\log_2 y) = 1 \\ \Rightarrow & \log_3 \left(\frac{\log_2 x}{\log_2 y} \right) = 1 \\ \Rightarrow & \frac{\log_2 x}{\log_2 y} = 3 \end{aligned}$$

$$\Rightarrow xy^3 = 1$$

$$\text{Also, } xy^2 = 9$$

$$\Rightarrow y = \frac{1}{9}$$

$$\therefore x = 729$$

$$2. a > 1, b > 1$$

$$2(\log_a c + \log_b c) = 9 \log_{ab} c$$

$$\Rightarrow 2 \left[\log c \left[\frac{\log b + \log a}{\log a \log b} \right] \right] = 9 \frac{\log c}{\log a + \log b}$$

$$\Rightarrow 2(\log a + \log b)^2 = 9(\log a)(\log b)$$

$$\Rightarrow 2(\log a)^2 + 2(\log b)^2 + 4(\log a)(\log b) = 9(\log a)(\log b)$$

$$\Rightarrow 2\log_b a + 2\log_a b + 4 = 9$$

$$\Rightarrow 2\log_b a + 2\log_a b = 5$$

$$\Rightarrow t + \frac{1}{t} = \frac{5}{2}, \text{ where } t = \log_a b$$

$$\Rightarrow 2t^2 - 5t + 2 = 0$$

$$\Rightarrow (2t - 1)(t - 2) = 0$$

$$\Rightarrow t = 1/2, t = 2$$

$$\Rightarrow \log_a b = 1/2 \text{ or } \log_a b = 2$$

$$3. \log_3 x \cdot \log_4 x \cdot \log_5 x = \log_3 x \cdot \log_4 x + \log_4 x \cdot \log_5 x + \log_5 x \cdot \log_3 x$$

$$\text{Let } \log_3 x = p, \log_4 x = q, \log_5 x = r$$

$$\Rightarrow \frac{1}{pqr} = \frac{1}{pq} + \frac{1}{qr} + \frac{1}{pr}$$

$$\Rightarrow p + q + r = 1$$

$$\Rightarrow \log_3 x + \log_4 x + \log_5 x = 1$$

$$\Rightarrow \log_x 60 = 1$$

$$\Rightarrow x = 60$$

$$4. \frac{3}{2} \log_4 (x+2)^2 + 3 = \log_4 (4-x)^3 + \log_4 (6+x)^3$$

$$\Rightarrow 3 = \log_4 \left(\frac{24 - 2x - x^2}{|x+2|} \right)^3$$

$$\Rightarrow \left(\frac{24 - 2x - x^2}{|x+2|} \right)^3 = 4^3$$

$$\Rightarrow \frac{24 - 2x - x^2}{|x+2|} = 4$$

$$\text{If } x+2 > 0$$

$$\Rightarrow x^2 + 6x - 16 = 0$$

$$\Rightarrow (x-2)(x+8) = 0$$

$$\Rightarrow x = 2$$

$$\text{If } x+2 < 0$$

$$x^2 - 2x - 32 = 0$$

$$\Rightarrow x = 1 - \sqrt{33}$$

$$5. \log_{3/4} \log_8 (x^2 + 7) + \log_{1/2} \log_{1/4} (x^2 + 7)^{-1} = -2$$

$$\Rightarrow \log_{3/4} \left(\frac{1}{3} \log_2 (x^2 + 7) \right) - \log_2 \frac{\log_2 (x^2 + 7)}{2} = -2$$

$$\text{Let } \log_2 (x^2 + 7) = t$$

$$\Rightarrow \log_{3/4} \frac{t}{3} - \log_2 \frac{t}{2} + 2 = 0$$

$$\Rightarrow \log_{3/4} \frac{t}{3} + 1 - \left(\log_2 \frac{t}{2} - 1 \right) = 0$$

$$\Rightarrow \log_{3/4} \frac{t}{4} = \log_2 \frac{t}{4}$$

$$\Rightarrow \frac{t}{4} = 1$$

$$\Rightarrow t = 4$$

$$\Rightarrow \log_2(x^2 + 7) = 4$$

$$\Rightarrow x^2 + 7 = 16$$

$$\Rightarrow x^2 = 9$$

$$\Rightarrow x = \pm 3$$

$$6. 5^{\log x} - 3^{\log x - 1} = 3^{\log x + 1} - 5^{\log x - 1}$$

$$\Rightarrow 5^{\log x} + 5^{\log x - 1} = 3^{\log x + 1} + 3^{\log x - 1}$$

$$\Rightarrow 5^{\log x} + \frac{5^{\log x}}{5} = 3 \cdot 3^{\log x} + \frac{3^{\log x}}{3}$$

$$\Rightarrow \frac{6 \cdot 5^{\log x}}{5} = \frac{10 \cdot 3^{\log x}}{3}$$

$$\Rightarrow \left(\frac{3}{5} \right)^{\log x} = \left(\frac{3}{5} \right)^2$$

$$\Rightarrow \log_{10} x = 2$$

$$\Rightarrow x = 100$$

7. Adding given equations

$$\log_a(xy z) [\log_a x + \log_a y + \log_a z] = 144$$

$$\Rightarrow \log_a(xy z) = (144)^{1/2} = 12$$

$$\Rightarrow xy z = a^{12}$$

$$\text{From } \log_a x \log_a(xy z) = 48$$

$$\Rightarrow (\log_a x)(12) = 48$$

$$\Rightarrow \log_a x = 4$$

$$\Rightarrow x = a^4$$

$$\text{Similarly } y = a \text{ and } z = a^7$$

$$8. \sqrt[4]{|x-3|^{x+1}} = \sqrt[3]{|x-3|^{x-2}}$$

Taking log on both the sides

$$\frac{x+1}{4} \log |x-3| = \frac{x-2}{3} \log |x-3|$$

$$\Rightarrow \log |x-3| \left[\frac{x+1}{4} - \frac{x-2}{3} \right] = 0$$

$$\Rightarrow \log |x-3| = 0 \text{ or } \left[\left(\frac{x+1}{4} \right) - \left(\frac{x-2}{3} \right) \right] = 0$$

$$\Rightarrow x = 4, 2 \text{ or } x = 11$$

$$9. \log_{x^2} 16 + \log_{2x} 64 = 3$$

$$\Rightarrow \frac{4}{\log_2 x^2} + \frac{6}{\log_2 2x} = 3$$

$$\Rightarrow \frac{2}{\log_2 x} + \frac{6}{1 + \log_2 x} = 3$$

$$\text{Put } \log_2 x = t$$

$$\Rightarrow \frac{2}{t} + \frac{6}{1+t} = 3$$

$$\Rightarrow 2 + 2t + 6t = 3t + 3t^2$$

$$\Rightarrow 3t^2 - 5t - 2 = 0$$

$$\Rightarrow (3t+1)(t-2) = 0$$

$$\Rightarrow t = -\frac{1}{3}, t = 2$$

$$\Rightarrow \log_2 x = -\frac{1}{3} \text{ or } \log_2 x = 2$$

$$\Rightarrow x = 2^{-1/3} \text{ or } x = 4$$

Single Correct Answer Type

$$10. (a) 5 \cdot 3^{\log_3 x} - 2^{1-\log_2 x} - 3 = 0$$

$$\Rightarrow 5x - \frac{2}{x} - 3 = 0$$

$$\Rightarrow 5x^2 - 3x - 2 = 0$$

$$\Rightarrow x = 1, x = -\frac{2}{5} \text{ (not possible)}$$

$$\Rightarrow x = 1$$

$$11. (c) 2 \log_{10} a + \log_{10}(a-1) = \log_{10} 2a$$

$$\therefore a^2(a-1) = 2a$$

$$\therefore a^2 - a - 2 = 0$$

$$\Rightarrow (a-2)(a+1) = 0$$

$$\Rightarrow a = 2, -1 \text{ (not possible)}$$

$$\therefore a = 2$$

$$12. (b) \text{ We have } 9^{\log_3(\log_e x)} = \log_e x - (\log_e x)^2 + 1$$

$$\therefore x > 1$$

$$\text{The given equation } 2(\log_e x)^2 - (\log_e x) - 1 = 0$$

$$\therefore \log_e x = \frac{-1}{2}, 1$$

$$\Rightarrow x = e, \frac{1}{\sqrt{e}} \text{ (not possible)}$$

$$\Rightarrow x = e$$

$$13. (b) x^{\log_x(1-x)^2} = 9$$

$$\Rightarrow (1-x)^2 = 9$$

$$\Rightarrow 1-x = \pm 3$$

$$\Rightarrow x = -2, 4$$

$$\text{But } x = -2 \text{ is not possible}$$

$$\Rightarrow x = 4$$

$$14. (c) \text{ We have}$$

$$(\sqrt{\pi})^{\log_{\pi} x} \cdot (\sqrt{\pi})^{\log_{\pi^2} x} \cdot (\sqrt{\pi})^{\log_{\pi^4} x} \cdot (\sqrt{\pi})^{\log_{\pi^8} x} \dots \infty = 3$$

$$\Rightarrow (\sqrt{\pi})^{\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots\right) \log_{\pi} x} = 3$$

$$\Rightarrow (\sqrt{\pi})^{2 \log_{\pi} x} = 3$$

$$\Rightarrow \pi^{\log_{\pi} x} = 3$$

$$\Rightarrow x = 3$$

$$15. (a) \text{ We have } 2^{\log_a(2x)} = 5^{\log_a(5x)}$$

Taking log on both sides, we get

$$\log_a(2x) \cdot \log 2 = \log_a(5x) \cdot \log 5$$

$$\Rightarrow \frac{(\log 2 + \log x)}{\log a} \log 2 = \frac{(\log 5 + \log x)}{\log a} \log 5$$

$$\Rightarrow (\log 2)^2 + \log x \log 2 = (\log 5)^2 + (\log x) \log 5$$

$$\Rightarrow \log x (\log 2 - \log 5) = (\log 5)^2 - (\log 2)^2$$

S.6 Trigonometry

$$\Rightarrow -\log x = \log 5 + \log 2 = \log 10$$

$$\Rightarrow x = \frac{1}{10}$$

$$16. (b) \log_5 \left(\log_{64} |x| + (25)^x - \frac{1}{2} \right) = 2x$$

$$\Rightarrow \log_{64} |x| + (25)^x - \frac{1}{2} = (25)^x$$

$$\Rightarrow \log_{64} |x| = \frac{1}{2}$$

$$\Rightarrow |x| = 8$$

$$\Rightarrow x = -8, 8$$

$$17. (c) \log_6 9 - \log_9 27 + \log_8 x = \log_{64} x - \log_6 4$$

$$\Rightarrow (\log_6 9 + \log_6 4) - \frac{3}{2} \log_3 3 = \frac{\log_8 x}{2} - \log_8 x$$

$$\Rightarrow 2 - \frac{3}{2} = -\frac{1}{2} \log_8 x$$

$$\Rightarrow \frac{1}{2} = -\frac{1}{2} \log_8 x$$

$$\Rightarrow x = \frac{1}{8}$$

DPP 1.4

Single Correct Answer Type

$$1. (c) \log_2(\log_2(\log_2 x)) = 2$$

$$\Rightarrow \log_2(\log_2 x) = 4$$

$$\Rightarrow \log_2 x = 16$$

$$\Rightarrow x = 2^{16}$$

$$\therefore \log_{10} x = 16 \log_{10} 2 = 16 \times 0.3010 = 4.8160$$

$$\therefore \text{Number of digits} = 5$$

$$2. (c) \log_{\sqrt{0.9}} \log_5(\sqrt{x^2 + 5 + x}) > 0$$

$$\Rightarrow 0 < \log_5(\sqrt{x^2 + 5 + x}) < 1$$

$$\Rightarrow 1 < (x^2 + 5 + x)^{1/2} < 5$$

$$\Rightarrow 1 < x^2 + 5 + x < 25$$

$$\Rightarrow x^2 + x - 20 < 0 \text{ (as } x^2 + x + 4 > 0 \text{ for all real } x)$$

$$\Rightarrow (x+5)(x-4) < 0$$

$$\Rightarrow x \in (-5, 4)$$

$$3. (b) \text{ Let } \log_2 x = t$$

$$\therefore \frac{1 - (t/2)}{1+t} \leq \frac{1}{2}$$

$$\Rightarrow \frac{2-t}{1+t} \leq 1$$

$$\Rightarrow \frac{2-t}{1+t} - 1 \leq 0$$

$$\Rightarrow \frac{2t-1}{t+1} \geq 0$$

$$\Rightarrow t < -1 \text{ or } t \geq \frac{1}{2}$$

$$\Rightarrow \log_2 x < -1 \text{ or } \log_2 x \geq \frac{1}{2}$$

$$\Rightarrow 0 < x < \frac{1}{2} \text{ or } x \geq \sqrt{2}$$

$$\Rightarrow \text{smallest integer is } 2$$

$$4. (c) \text{ We have } \log_9(x+1) \cdot \log_2(x+1) - \log_9(x+1) - \log_2(x+1) + 1 < 0$$

$$\Rightarrow (\log_9(x+1) - 1)(\log_2(x+1) - 1) < 0$$

$$\Rightarrow \left(\log_9 \left(\frac{x+1}{9} \right) \right) \left(\log_2 \left(\frac{x+1}{2} \right) \right) < 0$$

$$\text{Case I: } \log_9 \left(\frac{x+1}{9} \right) > 0$$

$$\Rightarrow \frac{x+1}{9} > 1$$

$$\Rightarrow x > 8$$

$$\therefore \log_2 \left(\frac{x+1}{2} \right) < 0$$

$$\Rightarrow 0 < \frac{x+1}{2} < 1$$

$$\Rightarrow -1 < x < 1$$

$$\therefore \text{from (1) and (2), we get } x \in \emptyset$$

$$\text{Case II: } \log_9 \left(\frac{x+1}{9} \right) < 0$$

$$\Rightarrow 0 < \frac{x+1}{9} < 1$$

$$\Rightarrow -1 < x < 8$$

$$\therefore \log_2 \left(\frac{x+1}{2} \right) > 0$$

$$\Rightarrow \frac{x+1}{2} > 1$$

$$\Rightarrow x > 1$$

$$\therefore \text{from (3) and (4), } x \in (1, 8)$$

$$\text{So integral values are } x = 2, 3, 4, 5, 6, 7$$

$$5. (a) \text{ Case I: } 1/x > 1 \text{ or } 0 < x < 1$$

$$\therefore \log_1 \left(\frac{2(x-2)}{(x+1)(x-5)} \right) \geq 1$$

$$\Rightarrow \frac{2(x-2)}{(x+1)(x-5)} \geq \frac{1}{x}$$

$$\Rightarrow \frac{2(x-2)}{(x+1)(x-5)} - \frac{1}{x} \geq 0$$

$$\Rightarrow \frac{2x(x-2) - (x+1)(x-5)}{x(x+1)(x-5)} \geq 0$$

$$\Rightarrow \frac{x^2 + 5}{x(x+1)(x-5)} \geq 0$$

$$\Rightarrow x \in (-1, 0) \cup (5, \infty)$$

Hence, no solution in this case.

Case II: $0 < \frac{1}{x} < 1$ or $x > 1$

$$\therefore 0 < x < 5$$

$$\text{Also } \frac{2(x-2)}{(x+1)(x-5)} > 0$$

$$\therefore x \in (1, 2)$$

(1)

Multiple Correct Answers Type

6. (a, c)

We have $\log_{1/2}(4-x) \geq \log_{1/2}2 - \log_{1/2}(x-1)$

It is defined if $4-x > 0$ and $x-1 > 0$ or $1 < x < 4$

$$\Rightarrow \log_{1/2}(4-x)(x-1) \geq \log_{1/2}2$$

$$\Rightarrow (4-x)(x-1) \leq 2$$

$$\Rightarrow x^2 - 5x + 6 \geq 0$$

$$\Rightarrow (x-3)(x-2) \geq 0$$

$$\Rightarrow x \geq 3 \text{ or } x \leq 2$$

From (i) and (ii), we get $x \in (1, 2] \cup [3, 4)$

(i)

(ii)

7. (a, b)

We have

$$2 \log_{\frac{1}{4}}(x+5) > \frac{9}{4} \log_{\frac{1}{3\sqrt{3}}}(9) + \log_{\sqrt{x+5}}(2)$$

$$\Rightarrow -\log_2(x+5) > \frac{9}{4} \left(-\frac{4}{3} \right) + \frac{2}{\log_2(x+5)}$$

$$\Rightarrow 3 > \log_2(x+5) + \frac{2}{\log_2(x+5)}$$

$$\text{Let } \log_2(x+5) = y$$

$$\Rightarrow 3 > y + \frac{2}{y}$$

$$\Rightarrow \frac{y^2+2}{y} - 3 < 0$$

$$\Rightarrow \frac{y^2-3y+2}{y} < 0$$

$$\Rightarrow \frac{(y-2)(y-1)}{y} < 0$$

Using sign scheme method, we get

$$y \in (-\infty, 0) \cup (1, 2)$$

$$\therefore \log_2(x+5) \in (-\infty, 0) \cup (1, 2)$$

$$\therefore (x+5) \in (2^{-\infty}, 2^0) \cup (2^1, 2^2)$$

$$\therefore (x+5) \in (0, 1) \cup (2, 4)$$

$$\therefore x \in (-5, -4) \cup (-3, -1)$$

8. (a, b)

$$\log_3 x - (\log_3 x)^2 \leq \frac{3}{2} \log_{(1/2\sqrt{2})} 4$$

$$\Rightarrow \log_3 x - (\log_3 x)^2 \leq -2$$

$$\Rightarrow (\log_3 x)^2 - \log_3 x - 2 \geq 0$$

$$\Rightarrow (\log_3 x - 2)(\log_3 x + 1) \geq 0$$

$$\Rightarrow \log_3 x \leq -1 \text{ or } \log_3 x \geq 2$$

$$\Rightarrow x \leq 1/3 \text{ or } x \geq 9$$

9. (b, c)

$$\text{Let } N_1 = 8^{12} \cdot 5^{35}$$

$$\Rightarrow \log_{10} N_1 = \log_{10} (2^{36} \cdot 5^{35})$$

$$= \log_{10} (10^{35} \cdot 2)$$

$$= 35 + \log_{10} 2$$

$$= 35 + 0.3010$$

$$= 35.3010$$

$\therefore$ Number of digits in N_1 is 36.

$$\text{Now, let } N_2 = \left(\frac{8}{27} \right)^{20}$$

$$\Rightarrow \log_{10} N_2 = 20 \log_{10} \left(\frac{8}{27} \right)$$

$$= 60 \log_{10} \frac{2}{3}$$

$$= 60 (\log_{10} 2 - \log_{10} 3)$$

$$= 60 \times (0.3010 - 0.4771)$$

$$= -60 \times 0.1761$$

$$= -10.56$$

$$= \overline{11} + 0.44$$

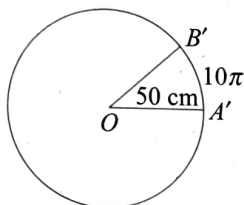
$\therefore$ Number of zeroes after decimal before a significant figure starts in N_2 is 10

CHAPTER 2

DPP 2.1

Single Correct Answer Type

1. (c) Length of wire = $2\pi r = 10\pi$



$$\therefore \text{Required angle} = \frac{10\pi}{50} = \frac{\pi}{5} \text{ radians}$$

2. (d) $\frac{\sin \theta}{1 - \cot \theta} + \frac{\cos \theta}{1 - \tan \theta}$

$$\begin{aligned} &= \frac{\sin \theta}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\cos \theta}{1 - \frac{\sin \theta}{\cos \theta}} \\ &= \frac{\sin^2 \theta}{\sin \theta - \cos \theta} + \frac{\cos^2 \theta}{\cos \theta - \sin \theta} \\ &= \frac{\sin^2 \theta - \cos^2 \theta}{(\sin \theta - \cos \theta)} = \sin \theta + \cos \theta \end{aligned}$$

3. (a) $\theta \in (\pi/4, \pi/2)$

$$\Rightarrow \tan \theta > 1 \Rightarrow 0 < \frac{1}{\tan \theta} < 1$$

$$\sum_{n=1}^{\infty} \frac{1}{\tan^n \theta} = \sin \theta + \cos \theta$$

$$\Rightarrow \frac{1}{\tan \theta} + \frac{1}{\tan^2 \theta} + \frac{1}{\tan^3 \theta} + \dots = \sin \theta + \cos \theta$$

$$\Rightarrow \frac{\frac{1}{\tan \theta}}{1 - \frac{1}{\tan \theta}} = \sin \theta + \cos \theta$$

$$\Rightarrow \frac{1}{\tan \theta - 1} = \sin \theta + \cos \theta$$

$$\Rightarrow \frac{\cos \theta}{\sin \theta - \cos \theta} = \sin \theta + \cos \theta$$

$$\Rightarrow \cos \theta = \sin^2 \theta - \cos^2 \theta = 1 - 2\cos^2 \theta$$

$$\Rightarrow 2\cos^2 \theta + \cos \theta - 1 = 0$$

$$\Rightarrow (2\cos \theta - 1)(\cos \theta + 1) = 0$$

$$\Rightarrow \cos \theta = \frac{1}{2} \text{ or } \cos \theta = -1 \text{ (rejected)}$$

$$\Rightarrow \theta = \frac{\pi}{3} \Rightarrow \tan \theta = \sqrt{3}$$

4. (b) $\tan^2 20^\circ - \sin^2 20^\circ = \tan^2 20^\circ (1 - \cos^2 20^\circ) = \tan^2 20^\circ \sin^2 20^\circ$

5. (d) $15 \sin^4 \alpha + 10 \cos^4 \alpha = 6$

Dividing by $\cos^4 \alpha$, we get

$$15 \tan^4 \alpha + 10 = 6 \sec^4 \alpha$$

$$\Rightarrow 15 \tan^4 \alpha + 10 = 6(1 + \tan^2 \alpha)^2$$

$$\Rightarrow 9 \tan^4 \alpha - 12 \tan^2 \alpha + 4 = 0$$

$$\Rightarrow (3 \tan^2 \alpha - 2)^2 = 0$$

$$\Rightarrow \tan^2 \alpha = \frac{2}{3}$$

Now $8 \operatorname{cosec}^6 \alpha + 27 \sec^6 \alpha$

$$= 8(1 + \cot^2 \alpha)^3 + 27(1 + \tan^2 \alpha)^3$$

$$= 8\left(1 + \frac{3}{2}\right)^3 + 27\left(1 + \frac{2}{3}\right)^3$$

$$= 125 + 125 = 250$$

6. (d) If the triangle is equilateral

$$\sin A + \sin B + \sin C = \frac{3\sqrt{3}}{2}$$

If the triangle is isosceles, let

$$A = 30^\circ, B = 30^\circ, C = 120^\circ. \text{ Then,}$$

$$\sin A + \sin B + \sin C = 1 + \frac{\sqrt{3}}{2}$$

If the triangle is right angled, let

$$A = 90^\circ, B = 30^\circ, C = 60^\circ. \text{ Then}$$

$$\sin A + \sin B + \sin C = \frac{3 + \sqrt{3}}{2}$$

If the triangle is right angled isosceles, then one of the angles is 90° and the remaining two are 45° each, so that

$$\sin A + \sin B + \sin C = 1 + \sqrt{2}$$

$$\text{and } \cos A + \cos B + \cos C = \sqrt{2}$$

7. (c) $\frac{\sin^2 x - 2 \cos^2 x + 1}{\sin^2 x + 2 \cos^2 x - 1} = 4$

$$\Rightarrow \sin^2 x - 2 \cos^2 x + 1$$

$$= 4 \sin^2 x + 8 \cos^2 x - 4$$

$$\Rightarrow 10 \cos^2 x + 3 \sin^2 x - 5 = 0$$

$$\Rightarrow 10 + 3 \tan^2 x - 5(1 + \tan^2 x) = 0$$

$$\Rightarrow 2 \tan^2 x = 5$$

$$\therefore \tan^2 x = \frac{5}{2}$$

8. (c) $\tan^2 \theta = \sin \theta \cos \theta$

$$\Rightarrow \sin \theta = \cos^3 \theta$$

Also the given expression is

$$\sin^6 \theta - 3 \sin^4 \theta + 3 \sin^2 \theta - 1 + 1 + \sin^2 \theta$$

$$= (\sin^2 \theta - 1)^3 + 1 + \sin^2 \theta$$

$$= -(1 - \sin^2 \theta)^3 + 1 + \sin^2 \theta$$

$$= -\cos^6 \theta + 1 + \sin^2 \theta$$

$$= -\sin^2 \theta + 1 + \sin^2 \theta = 1$$

9. (d) Given that $\tan \theta - \cot \theta = a$

and $\sin \theta + \cos \theta = b$

Now $(b^2 - 1)^2(a^2 + 4)$

$$= \{(\sin \theta + \cos \theta)^2 - 1\}^2 \{\tan \theta - \cot \theta\}^2 + 4\}$$

$$= [1 + 2 \sin \theta \cdot \cos \theta - 1]^2 (\tan^2 \theta + \cot^2 \theta - 2 + 4)$$

$$= 4 \sin^2 \theta \cdot \cos^2 \theta (\operatorname{cosec}^2 \theta + \sec^2 \theta)$$

$$= 4 \sin^2 \theta \cos^2 \theta \left[\frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} \right] = 4$$

10. (d) $18 \sin^2 \theta + 2 \operatorname{cosec}^2 \theta - 3$

$$= (\sqrt{18} \sin \theta - \sqrt{2} \operatorname{cosec} \theta)^2 + 9$$

Hence, the least value is 9.

11. (c) $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$

$$= \frac{\tan^2 \alpha}{1 + \tan^2 \alpha} + \frac{\tan^2 \beta}{1 + \tan^2 \beta} + \frac{\tan^2 \gamma}{1 + \tan^2 \gamma}$$

$$= \frac{x}{1+x} + \frac{y}{1+y} + \frac{z}{1+z} \quad [\text{where } x = \tan^2 \alpha, y = \tan^2 \beta, z = \tan^2 \gamma]$$

$$= \frac{(x+y+z) + (xy+yz+zx+2xyz) + xy+yz+zx+xyz}{(1+x)(1+y)(1+z)}$$

$$= \frac{1+x+y+z+xy+yz+zx+xyz}{(1+x)(1+y)(1+z)} = 1$$

$$[\because xy+yz+zx+2xyz=1]$$

12. (c) $E = \tan x (\cot y + \cot z) + \tan y (\cot z + \cot x) + \tan z (\cot x + \cot y)$

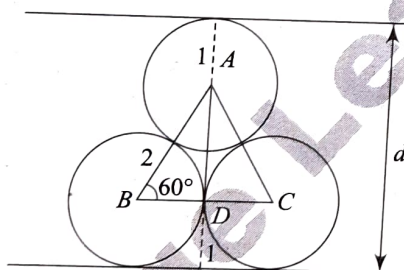
Using A.M. $\geq$ G.M.

$$\frac{\tan x \cot y + \tan x \cot z + \tan y \cot z + \tan y \cot x + \tan z \cot x + \tan z \cot y}{6}$$

$$\geq \frac{(\tan x \cdot \cot y \cdot \tan x \cdot \cot z \cdot \tan y \cdot \cot z \cdot \tan y \cdot \cot x \cdot \tan z \cdot \cot x \cdot \tan z \cdot \cot y)}{6}$$

$$\therefore E \geq 6$$

13. (a)



$$\sin 60^\circ = \frac{AD}{2}$$

$$\therefore AD = 2 \sin 60^\circ = \frac{2\sqrt{3}}{2}$$

$$\therefore d = 1 + AD + 1 = 2 + \sqrt{3}$$

14. (b) α and β are the roots of the equation $\frac{\cos \theta}{\cos A} + \frac{\sin \theta}{\sin A} = 1$

$$\Rightarrow \left(\frac{\cos \theta}{\cos A} \right)^2 = \left(1 - \frac{\sin \theta}{\sin A} \right)^2$$

$$\Rightarrow \frac{1 - \sin^2 \theta}{\cos^2 A} = 1 - 2 \frac{\sin \theta}{\sin A} + \left(\frac{\sin \theta}{\sin A} \right)^2$$

(1)

(2)

$$\Rightarrow \frac{1}{\cos^2 A} = 1 - 2 \frac{\sin \theta}{\sin A} + \frac{\sin^2 \theta}{\sin^2 A} + \frac{\sin^2 \theta}{\cos^2 A}$$

$$\Rightarrow 1 - \frac{1}{\cos^2 A} - 2 \frac{\sin \theta}{\sin A} + \frac{\sin^2 \theta}{\sin^2 A \cos^2 A} = 0$$

$$\Rightarrow -\frac{\sin^2 A}{\cos^2 A} - 2 \frac{\sin \theta}{\sin A} + \frac{\sin^2 \theta}{\sin^2 A \cos^2 A} = 0$$

$$\Rightarrow -\frac{\sin^2 A}{\cos^2 A} - 2 \frac{\sin \theta}{\sin A} + \frac{\sin^2 \theta}{\sin^2 A \cos^2 A} = 0$$

$$\Rightarrow \left(\frac{\sin \theta}{\sin A} \right)^2 - 2 \left(\frac{\sin \theta}{\sin A} \right) \cos^2 A - \sin^2 A = 0$$

Clearly,

$\frac{\sin \alpha}{\sin A}$ and $\frac{\sin \beta}{\sin A}$ are the roots of this equation

$$\therefore \frac{\sin \alpha \sin \beta}{\sin A \sin A} = -\sin^2 A$$

$$\Rightarrow \frac{\sin \alpha \sin \beta}{\sin^2 A} = -\sin^2 A$$

(1)

Similarly, by making a quadratic in $\frac{\cos \theta}{\cos A}$, we get

$$\frac{\cos \alpha \cos \beta}{\cos^2 A} = -\cos^2 A$$

(2)

Adding (1) and (2), we get

$$\frac{\cos \alpha \cos \beta}{\cos^2 A} + \frac{\sin \alpha \sin \beta}{\sin^2 A} = -(\cos^2 A + \sin^2 A) = -1$$

DPP 2.2

Single Correct Answer Type

1. (c) Let $m = 2k$, i.e. m is even where $k \in \mathbb{I}$

$$\therefore \beta = 2k\pi + \frac{\pi}{2} - A = \left(2k + \frac{1}{2} \right) \pi - A \quad (1)$$

If $m = 2k + 1$, i.e. m is odd, then

$$\therefore \beta = (2k + 1) \pi - \left(\frac{\pi}{2} - A \right) = \left(2k + \frac{1}{2} \right) \pi + A \quad (2)$$

From (1) and (2), β can be expressed as

$$\beta = \left(2k + \frac{1}{2} \right) \pi \pm A, k \in \mathbb{I}$$

which is same as α .

2. (d) $\sin 765^\circ = \sin(720^\circ + 45^\circ) = \sin 45^\circ = \frac{1}{\sqrt{2}}$

$$\cot \left(-\frac{15\pi}{4} \right) = -\cot \left(\frac{15\pi}{4} \right)$$

$$= -\cot \left(4\pi - \frac{\pi}{4} \right)$$

$$= \cot \frac{\pi}{4} = 1$$

S.10 Trigonometry

$$\tan \frac{13\pi}{3} = \tan \left(4\pi + \frac{\pi}{3} \right) = \tan \frac{\pi}{3} = \sqrt{3}$$

$$\begin{aligned} \operatorname{cosec}(-1410^\circ) &= -\operatorname{cosec}(1410^\circ) \\ &= -\operatorname{cosec}(1440^\circ - 30^\circ) \\ &= \operatorname{cosec}(30^\circ) = 2 \end{aligned}$$

3. (d) $5 \cos A + 3 = 0$

$$\Rightarrow \cos A = -3/5$$

$$\Rightarrow \sin A = 4/5$$

$$\Rightarrow \tan A = -4/3$$

Now equation whose roots are $4/5$ and $-4/3$ is

$$(x - 4/5)(x + 4/3) = 0$$

$$\text{or } 15x^2 + 8x - 16 = 0$$

4. (b) $\tan 1 > \cot 1$, also $\tan 1 > \tan \pi/4$ or $\tan 1 > 1$

$$\Rightarrow \tan^2 1 \text{ is greatest}$$

5. (c) $\sin y = x^2 - 2x = (x - 1)^2 - 1 \geq -1$

$$\text{But } \sin y \in [-1, 1].$$

Hence, infinite values of x are possible.

6. (d) $\sin 1 > \cos 1$ (as $1 > \pi/4$ for which $\sin x > \cos x$)

$$\Rightarrow \cos(\sin 1) < \cos(\cos 1) \text{ (as } \cos x \text{ is decreasing in the first quadrant)}$$

$$\text{Also } \sin(\sin 1) > \sin(\cos 1) \text{ (as } \sin x \text{ is increasing in the first quadrant)}$$

$$\text{Also } \cos 2 < 0; \text{ hence, } \cos 2 \in \text{IV quadrant, where } \cos x \text{ is +ve and } \sin x \text{ is -ve.}$$

$$\text{Hence, } \cos(\cos 2) > 0 \text{ and } \sin(\cos 2) < 0$$

$$\Rightarrow \cos(\cos 2) > \sin(\cos 2)$$

7. (c) $\sin^4 \alpha + \cos^4 \beta + 2 = 4 \sin \alpha \cos \beta$

$$\Rightarrow (\sin^2 \alpha - 1)^2 + (\cos^2 \beta - 1)^2 + 2(\sin \alpha - \cos \beta)^2 = 0$$

$$\Rightarrow \sin^2 \alpha = 1, \cos^2 \beta = 1 \text{ and } \sin \alpha = \cos \beta$$

$$\text{Now } 0 \leq \alpha, \beta \leq \frac{\pi}{2}$$

$$\therefore \sin \alpha = 1 \text{ and } \cos \beta = 1$$

$$\text{Hence, } (\sin \alpha + \cos \beta) = 1 + 1 = 2$$

8. (c) We have $\sec^2(a + 2)x = 1 - a^2$

$$\text{Now } \sec^2 x \geq 1$$

$$\Rightarrow 1 - a^2 \geq 1$$

$$\Rightarrow a = 0$$

$$\text{So, } \sec^2(a + 2)x = 1$$

$$\Rightarrow \sec^2 2x = 1$$

$$\Rightarrow \cos^2 2x = 1$$

$$\Rightarrow x = -\frac{\pi}{2}, 0, \frac{\pi}{2}$$

9. (b) $y = \sin^3 x - 6 \sin^2 x + 11 \sin x - 6$

$$\text{Put } \sin x = t$$

$$\therefore y = t^3 - 6t^2 + 11t - 6, -1 \leq t \leq 1$$

$$\therefore \frac{dy}{dt} = 3t^2 - 12t + 11 = 3(t - 2)^2 - 1 > 0 \text{ for } -1 \leq t \leq 1$$

$$\therefore \text{Function is increasing}$$

$$\text{Hence, range is } [f(-1), f(1)] \text{ or } [-24, 0]$$

10. (a) $a \sin x + c < 0, \forall x \in R$

$$\Rightarrow \sin x < -\frac{c}{a}$$

$$\Rightarrow -\frac{c}{a} > \sin x, \forall x \in R$$

$$\Rightarrow -\frac{c}{a} > 1$$

$$\Rightarrow -c > a$$

$$\Rightarrow c < -a$$

11. (b) $y = \sin^2 x + 8 \cos x - 7$

$$1 - \cos^2 x + 8 \cos x - 7$$

$$= -[\cos^2 x - 8 \cos x + 6]$$

$$= -[(\cos x - 4)^2 - 10]$$

$$= 10 - (4 - \cos x)^2$$

$$\therefore y_{\min} = 10 - 16 = -6$$

$$y_{\max} = 10 - 9 = 1$$

12. (c) $\tan \theta_1 = \cos \theta_1 < 1$

$$\therefore \tan \theta_1 < \tan \pi/4$$

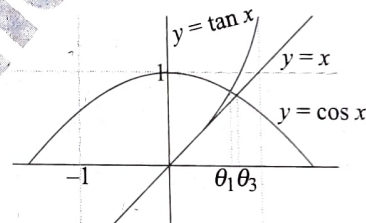
$$\therefore \theta_1 < 45^\circ$$

$$\tan \theta_2 = \operatorname{cosec} \theta_2 > 1$$

$$\therefore \tan \theta_2 > \tan 45^\circ$$

$$\therefore \theta_2 > 45^\circ$$

For $\cos \theta_3 = \theta_3$, draw the graphs of $y = \cos x$ and $y = x$



From the graph, $\theta_1 < \theta_3$

$$\theta_3 < 45^\circ$$

Thus, $\theta_1 < \theta_3 < \theta_2$

DPP 2.3

Single Correct Answer Type

1. (c) $2 \cos 10^\circ + \sin 100^\circ + \sin 1000^\circ + \sin 10000^\circ$

$$= 2 \cos 10^\circ + \sin(90^\circ + 10^\circ)$$

$$+ \sin(3 \times 360^\circ - 80^\circ) + \sin(28 \times 360^\circ - 80^\circ)$$

$$= 2 \cos 10^\circ + \cos 10^\circ - \sin 80^\circ - \sin 80^\circ$$

$$= 3 \cos 10^\circ - 2 \sin 80^\circ$$

$$= \cos 10^\circ$$

2. (c) $a = \sin \theta + \sin \left(\frac{3\pi}{2} - \theta \right)$

$$= \sin \theta - \cos \theta$$

$$b = \cos \theta - \cos \left(\frac{3\pi}{2} - \theta \right)$$

$$= \cos \theta + \sin \theta$$

$$a^2 + b^2 = (\sin \theta - \cos \theta)^2 + (\sin \theta + \cos \theta)^2 = 2$$

$$\therefore \text{Hypotenuse} = c = \sqrt{2}$$

3. (d) $\angle A = \pi - C$

$$\Rightarrow \tan A = -\tan C$$

$$\Rightarrow \tan A \cot C = -1$$

$$\angle B = \pi - D$$

$$\Rightarrow \sec B = -\sec D$$

$$\Rightarrow \sec B \cos D = -1$$

$$\Rightarrow \angle B = \pi - D$$

$$\Rightarrow \operatorname{cosec} B = \operatorname{cosec} D$$

$$\Rightarrow \operatorname{cosec} B \sin D = 1$$

$$4. (b) \quad x = \sin 130^\circ \cos 80^\circ, y = \sin 80^\circ \cos 130^\circ$$

$$\Rightarrow x = \cos 40^\circ \cos 80^\circ, y = -\sin 80^\circ \sin 40^\circ$$

$$\text{So, } x > 0 \text{ and } y < 0 \text{ and } xy < 0$$

$$\text{Now, } z = 1 + xy \Rightarrow 0 < z < 1.$$

$$5. (c) \quad \text{Since } \cos A + \cos B = 0$$

$$\Rightarrow A + B = \pi$$

$$\therefore B = \pi - A$$

$$\therefore \text{from } \sin A + \sin B = 1$$

$$\sin A + \sin(\pi - A) = 1$$

$$\Rightarrow \sin A = \frac{1}{2}$$

$$\therefore A = 30^\circ \text{ and } B = 150^\circ$$

$$\text{or } A = 150^\circ \text{ and } B = 30^\circ$$

$$\therefore 12 \cos 60^\circ + 4 \cos 300^\circ = 8$$

$$6. (a) \quad \sin \frac{\pi}{9} + \sin \frac{2\pi}{9} + \sin \frac{3\pi}{9} + \dots + \sin \frac{17\pi}{9}$$

$$= \left(\sin \frac{\pi}{9} + \sin \frac{17\pi}{9} \right)$$

$$+ \left(\sin \frac{2\pi}{9} + \sin \frac{16\pi}{9} \right)$$

...

...

$$+ \left(\sin \frac{8\pi}{9} + \sin \frac{10\pi}{9} \right)$$

$$+ \sin \frac{9\pi}{9}$$

$$= 0$$

$$7. (c) \quad \cos^2 73^\circ + \cos^2 47^\circ - \sin^2 43^\circ + \sin^2 107^\circ$$

$$= \cos^2 73^\circ + \cos^2 47^\circ - \sin^2 (90^\circ - 47^\circ) + \sin^2 (180^\circ - 73^\circ)$$

$$= \cos^2 73^\circ + \cos^2 47^\circ - \cos^2 47^\circ + \sin^2 73^\circ$$

$$= 1$$

$$8. (b) \quad \frac{\tan \left(x - \frac{\pi}{2} \right) \cdot \cos \left(\frac{3\pi}{2} + x \right) - \sin^3 \left(\frac{7\pi}{2} - x \right)}{\cos \left(x - \frac{\pi}{2} \right) \cdot \tan \left(\frac{3\pi}{2} + x \right)}$$

$$= \frac{(-\cot)(\sin x) + \cos^3 x}{(\sin x) \cdot (-\cot x)}$$

$$= \frac{-\cos x + \cos^3 x}{-\cos x}$$

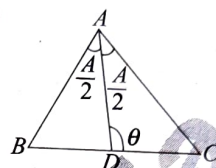
$$= 1 - \cos^2 x$$

$$= \sin^2 x$$

9. (c) Given expression

$$\begin{aligned} &= \frac{\sin(360^\circ - 60^\circ) \cdot \tan(360^\circ - 30^\circ) \cdot \sec(360^\circ + 60^\circ)}{\tan(180^\circ - 45^\circ) \cdot \sin(180^\circ + 30^\circ) \cdot \sec(360^\circ - 45^\circ)} \\ &= \frac{(-\sin 60^\circ) \cdot (-\tan 30^\circ) \cdot \sec 60^\circ}{(-\tan 45^\circ) \cdot (-\sin 30^\circ) \cdot \sec 45^\circ} \\ &= \frac{\sin 60^\circ \cdot \tan 30^\circ \cdot \sec 60^\circ}{\tan 45^\circ \cdot \sin 30^\circ \cdot \sec 45^\circ} \\ &= \frac{\frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{3}} \cdot 2}{1 \cdot \frac{1}{2} \cdot \sqrt{2}} = \sqrt{2} \end{aligned}$$

10. (c)



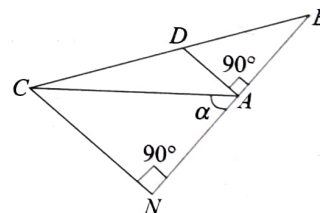
$$\theta = \pi - \left(C + \frac{A}{2} \right)$$

$$\Rightarrow \sin \theta = \sin \left(C + \frac{A}{2} \right)$$

$$= \sin \left(B + \frac{\pi}{2} - \frac{B+C}{2} \right)$$

$$= \cos \left(\frac{B-C}{2} \right)$$

11. (c) We have $BD = DC$ and $\angle DAB = 90^\circ$. Draw CN perpendicular to BA produced, then in $\triangle BCN$, we have $DA = \frac{1}{2}CN$ and $AB = AN$



Let $\angle CAN = \alpha$

$$\therefore \tan A = \tan(\pi - \alpha)$$

$$= -\tan \alpha$$

$$= -\frac{CN}{NA}$$

$$= -2 \frac{AD}{AB} \quad (\because \triangle BAD \sim \triangle BNC \text{ and } AC = 2BD)$$

$$= -2 \tan B$$

$$\Rightarrow \tan A + 2 \tan B = 0$$

$$12. (c) \quad f(\theta) = 1 + \sin \left(\frac{\pi}{6} + \theta \right) + 2 \cos \left(\frac{\pi}{3} - \theta \right)$$

$$= 1 + \sin \left(\frac{\pi}{6} + \theta \right) + 2 \sin \left(\frac{\pi}{2} - \left(\frac{\pi}{3} - \theta \right) \right)$$

$$= 1 + \sin \left(\frac{\pi}{6} + \theta \right) + 2 \sin \left(\frac{\pi}{6} + \theta \right)$$

$$= 1 + 3 \sin \left(\frac{\pi}{6} + \theta \right)$$

$\therefore$ Maximum value of $f(\theta)$ is 4.

CHAPTER 3

DPP 3.1

Single Correct Answer Type

1. (d) $a_1 \cos(\alpha_1 + \theta) + a_2 \cos(\alpha_2 + \theta) + \dots + a_n \cos(\alpha_n + \theta) = 0$
 $\Rightarrow (a_1 \cos \alpha_1 + a_2 \cos \alpha_2 + \dots + a_n \cos \alpha_n) \cos \theta$
 $\quad - (a_1 \sin \alpha_1 + a_2 \sin \alpha_2 + \dots + a_n \sin \alpha_n) \sin \theta = 0$
 $\Rightarrow a_1 \sin \alpha_1 + a_2 \sin \alpha_2 + \dots + a_n \sin \alpha_n = 0$
 (since $\sin \theta \neq 0$)
 and $a_1 \cos \alpha_1 + a_2 \cos \alpha_2 + \dots + a_n \cos \alpha_n = 0$
 Now, $a_1 \cos(\alpha_1 + \lambda) + a_2 \cos(\alpha_2 + \lambda) + \dots + a_n \cos(\alpha_n + \lambda)$
 $= (a_1 \cos \alpha_1 + a_2 \cos \alpha_2 + \dots + a_n \cos \alpha_n) \cos \lambda$
 $\quad - (a_1 \sin \alpha_1 + a_2 \sin \alpha_2 + \dots + a_n \sin \alpha_n) \sin \lambda = 0$

2. (b) $\frac{\cos^2 33^\circ - \cos^2 57^\circ}{\cos 69^\circ - \cos 21^\circ}$
 $= \frac{\sin(57^\circ + 33^\circ) \sin(57^\circ - 33^\circ)}{-2 \sin \frac{69^\circ + 21^\circ}{2} \sin \frac{69^\circ - 21^\circ}{2}}$
 $= \frac{\sin 90^\circ \times \sin 24^\circ}{-2 \sin 45^\circ \sin 24^\circ}$
 $= \frac{1}{-2 \times \frac{1}{\sqrt{2}}} = -\frac{1}{\sqrt{2}}$

3. (b) $\frac{(2 \sin 50^\circ \cdot \sin 40^\circ) \sin 10^\circ}{\sin 10^\circ \cdot \cos 10^\circ}$
 $= \frac{(\cos 10^\circ - \cos 90^\circ) \sin 10^\circ}{\sin 10^\circ \cdot \cos 10^\circ}$
 $= 1$

4. (a) $\cos 65^\circ \cos 55^\circ \cos 5^\circ$
 $= \left(\frac{\cos 60^\circ + \cos 50^\circ}{2} \right) \cdot \cos 65^\circ$
 $= \frac{\cos 65^\circ}{4} + \frac{2 \cos 50^\circ \cos 65^\circ}{4}$
 $= \frac{\cos 65^\circ}{4} + \frac{\cos 115^\circ + \cos 15^\circ}{4}$
 $= \frac{2 \cos 90^\circ \cos 25^\circ + \cos 15^\circ}{4}$
 $= \frac{\cos 15^\circ}{4} = \frac{\sqrt{3} + 1}{8\sqrt{2}}$

5. (a) Squaring and adding, we get
 $x^2 + y^2 = 1 + 1 + 2 \cos(2A - A)$
 $\therefore \frac{x^2 + y^2 - 2}{2} = \cos A$

Also $\cos A + 2 \cos^2 A - 1 = y$
 or $(\cos A + 1)(2 \cos A - 1) = y$
 Put value of $\cos A$ from (1) and get the answer.

6. (a) $\sin \theta + \sin 3\theta + \sin 2\theta = \sin \alpha$
 $\Rightarrow 2 \sin 2\theta \cos \theta + \sin 2\theta = \sin \alpha$

$$\Rightarrow \sin 2\theta(2 \cos \theta + 1) = \sin \alpha \quad (1)$$

$$\text{Now } \cos \theta + \cos 3\theta + \cos 2\theta = \cos \alpha$$

$$2 \cos 2\theta \cos \theta + \cos 2\theta = \cos \alpha$$

$$\cos 2\theta(2 \cos \theta + 1) = \cos \alpha \quad (2)$$

From (1) and (2),

$$\tan 2\theta = \tan \alpha \Rightarrow 2\theta = \alpha \Rightarrow \theta = \alpha/2$$

7. (a) $\frac{\sin 22^\circ \cos 8^\circ + \cos 158^\circ \cos 98^\circ}{\sin 23^\circ \cos 7^\circ + \cos 157^\circ \cos 97^\circ}$
 $= \frac{\sin 22^\circ \cos 8^\circ + \cos(180^\circ - 22^\circ) \cos(90^\circ + 8^\circ)}{\sin 23^\circ \cos 7^\circ + \cos(180^\circ - 23^\circ) \cos(90^\circ + 7^\circ)}$
 $= \frac{\sin 22^\circ \cos 8^\circ + \cos 22^\circ \sin 8^\circ}{\sin 23^\circ \cos 7^\circ + \cos 23^\circ \sin 7^\circ}$
 $= \frac{\sin 30^\circ}{\sin 30^\circ} = 1$

8. (b) $y = 4 \sin \frac{\alpha}{2} \sin \frac{\beta}{2} \cos \left(\frac{\alpha + \beta}{2} \right)$
 $= 2 \left[\cos \frac{\alpha - \beta}{2} - \cos \frac{\alpha + \beta}{2} \right] \cos \left(\frac{\alpha + \beta}{2} \right)$
 $= \cos \alpha + \cos \beta - (1 + \cos(\alpha + \beta)) = x - 1$
 $\Rightarrow x - y = 1$

9. (c) $\frac{\cos 5^\circ \cos 20^\circ + \cos 35^\circ \cos 50^\circ}{\sin 5^\circ \cos 20^\circ - \sin 35^\circ \cos 50^\circ}$
 $\quad + \cos 5^\circ \sin 20^\circ - \cos 35^\circ \sin 50^\circ$
 $= \frac{(\cos 25^\circ + \cos 15^\circ) + (\cos 85^\circ + \cos 15^\circ)}{(\sin 25^\circ - \sin 15^\circ) - (\sin 85^\circ - \sin 15^\circ)}$
 $= \frac{\cos 25^\circ + \cos 85^\circ}{\sin 25^\circ - \sin 85^\circ}$
 $= \frac{2 \cos 55^\circ \cos 30^\circ}{2 \cos 55^\circ \sin 30^\circ}$
 $= -\cot 30^\circ = -\sqrt{3}$

10. (c) $\sec 11^\circ \cdot \sec 19^\circ - 2 \cot 71^\circ$
 $= \frac{1}{\cos 11^\circ \cos 19^\circ} - \frac{2 \sin 19^\circ}{\cos 19^\circ}$
 $= \frac{1 - 2 \sin 19^\circ \cos 11^\circ}{\cos 11^\circ \cos 19^\circ}$
 $= \frac{1 - (\sin 30^\circ + \sin 8^\circ)}{\cos 11^\circ \cos 19^\circ}$
 $= \frac{1 - (1/2) - \sin 8^\circ}{\cos 11^\circ \cos 19^\circ}$
 $= \frac{\sin 30^\circ - \sin 8^\circ}{\cos 11^\circ \cos 19^\circ}$
 $= \frac{2 \sin 11^\circ \cos 19^\circ}{\cos 11^\circ \cos 19^\circ}$
 $= 2 \tan 11^\circ$

11. (c) Since $\sin \alpha + \sin \beta + \sin \gamma > \sin(\alpha + \beta + \gamma)$

when $\alpha + \beta + \gamma = \pi$

$$\therefore \sin \alpha + \sin \beta + \sin \gamma > 0$$

$\therefore$ The minimum value of $\sin \alpha + \sin \beta + \sin \gamma$ is always positive.

12. (b) We have $\sin \alpha = 1/\sqrt{5} \Rightarrow \cos \alpha = 2/\sqrt{5}$

$$\text{and } \sin \beta = 3/5 \Rightarrow \cos \beta = 4/5$$

$$\sin(\beta - \alpha) = \sin \beta \cos \alpha - \sin \alpha \cos \beta$$

$$= \frac{3}{5} \cdot \frac{2}{\sqrt{5}} - \frac{1}{\sqrt{5}} \cdot \frac{4}{5} = \frac{2}{5\sqrt{5}} = 0.1789$$

$$\text{Now } \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} = 0.7071 = \sin \frac{3\pi}{4}$$

$$\text{Since } 0 < 0.1789 < 0.7071$$

$$\therefore \sin 0 < \sin(\beta - \alpha) < \sin \frac{\pi}{4} \Rightarrow 0 < (\beta - \alpha) < \frac{\pi}{4}$$

$$\text{Also, } \sin \pi < \sin(\beta - \alpha) < \sin \frac{3\pi}{4}$$

$$\therefore (\beta - \alpha) \in [0, \pi/4] \text{ or } [3\pi/4, \pi]$$

13. (a) $\tan A + \tan B + \tan C$

$$= \tan A \tan B \tan C = 6$$

$$\therefore \cot A \cot B \cot C = 1/6$$

14. (b) $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$

Squaring and adding, we get

$$2 + 2 \cos(\alpha - \beta) = 0$$

$$\therefore \cos(\alpha - \beta) = -1$$

$$\therefore \cos 2\alpha + \cos 2\beta$$

$$= 2 \cos(\alpha + \beta) \cos(\alpha - \beta)$$

$$= -2 \cos(\alpha + \beta)$$

15. (c) $\tan^2 \theta - 4 \tan \theta + 1 = 0$

$$\Rightarrow \tan \theta = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$$

$$\Rightarrow \tan(\alpha + \beta) = 2 + \sqrt{3} \text{ and } \tan(\alpha - \beta) = 2 - \sqrt{3}$$

$$\Rightarrow \alpha + \beta = 75^\circ \text{ and } \alpha - \beta = 15^\circ$$

$$\Rightarrow \alpha = 45^\circ \text{ and } \beta = 30^\circ$$

16. (d) $\sin \alpha + \sin \beta + \sin \gamma \geq 2$

$$\text{or } \Sigma \sin \alpha \geq 2$$

Squaring both sides, we get

$$\Sigma \sin^2 \alpha + 2 \Sigma \sin \alpha \sin \beta \geq 4$$

$$\Rightarrow 3 - \Sigma \cos^2 \alpha + 2 \Sigma \sin \alpha \sin \beta \geq 4$$

$$\Rightarrow 2 \Sigma \sin \alpha \sin \beta \geq 1 + \Sigma \cos^2 \alpha$$

$$\Rightarrow 2 \Sigma (\sin \alpha \sin \beta + \cos \alpha \cos \beta)$$

$$\geq 1 + (\Sigma \cos \alpha)^2 \text{ (adding } 2 \Sigma \cos \alpha \cos \beta \text{ on both sides)}$$

$$\Rightarrow (\cos \alpha + \cos \beta + \cos \gamma)^2 + 1$$

$$\leq 2[\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha)] \leq 6$$

$$\Rightarrow (\cos \alpha + \cos \beta + \cos \gamma)^2 \leq 5$$

$$\Rightarrow \cos \alpha + \cos \beta + \cos \gamma \leq \sqrt{5}$$

17. (b) Clearly, $2 \sin \theta + 3 \cos \theta + 4 > 0 \forall \theta \in R$

$$\text{and } 6 - 2 \sin \theta - 3 \cos \theta > 0 \forall \theta \in R$$

$$\text{Put, } 2 \sin \theta + 3 \cos \theta + 4 = x \text{ and } 6 - 2 \sin \theta - 3 \cos \theta = y$$

$$\text{Now, } x + y = 10$$

Also, A.M $\geq$ G.M. (for positive numbers)

$$\therefore \frac{\frac{x}{3} + \frac{x}{3} + \frac{x}{3} + \frac{y}{2} + \frac{y}{2}}{5} \geq \left(\frac{x^3}{27} \cdot \frac{y^2}{4} \right)^{\frac{1}{5}}$$

$$\therefore (2 \sin \theta + 3 \cos \theta + 4)^3 (6 - 2 \sin \theta - 3 \cos \theta)^2 \geq (32)(27)(4) = 3456$$

18. (c) $\sin x \sin y + 3 \cos y + 4 \sin y \cos x = \sqrt{26}$

$$\therefore 3 \cos y + (\sin x + 4 \cos x) \sin y = \sqrt{26}$$

$$\therefore 3 \cos y + (\sin x + 4 \cos x) \sin y$$

$$\leq \sqrt{9 + (\sin x + 4 \cos x)^2}$$

$$\leq \sqrt{9 + 1 + 16}$$

$$= \sqrt{26}$$

$$\therefore \sin x \sin y + 3 \cos y + 4 \sin y \cos x = \sqrt{26}$$

$$\Rightarrow \sin x \sin y = \frac{\cos y}{3} - \frac{\sin y \cos x}{4}$$

$$\Rightarrow 3 \tan y = \operatorname{cosec} x \text{ and } \tan x = 1/4$$

$$\Rightarrow 9 \tan^2 y = \operatorname{cosec}^2 x = (1 + \cot^2 x) = 17$$

$$\Rightarrow \tan^2 x + \cot^2 y = \frac{1}{16} + \frac{9}{17}$$

19. (b) From the third relation, we get

$$\cos \theta \cos \phi + \sin \theta \sin \phi = \sin \beta \sin \gamma$$

$$\Rightarrow \sin^2 \theta \sin^2 \phi = (\cos \theta \cos \phi - \sin \beta \sin \gamma)^2$$

$$\Rightarrow \left(1 - \frac{\sin^2 \beta}{\sin^2 \alpha}\right) \left(1 - \frac{\sin^2 \gamma}{\sin^2 \alpha}\right) = \left(\frac{\sin \beta \sin \gamma}{\sin^2 \alpha} - \sin \beta \sin \gamma\right)^2$$

$$\Rightarrow (\sin^2 \alpha - \sin^2 \beta)(\sin^2 \alpha - \sin^2 \gamma) = \sin^2 \beta \sin^2 \gamma (1 - \sin^2 \alpha)^2$$

$$\Rightarrow \sin^4 \alpha (1 - \sin^2 \beta \sin^2 \gamma) - \sin^2 \alpha (\sin^2 \beta + \sin^2 \gamma - 2 \sin^2 \beta \sin^2 \gamma) = 0$$

$$\therefore \sin^2 \alpha = \frac{\sin^2 \beta + \sin^2 \gamma - 2 \sin^2 \beta \sin^2 \gamma}{1 - \sin^2 \beta \sin^2 \gamma}$$

$$\text{and } \cos^2 \alpha = \frac{1 - \sin^2 \beta - \sin^2 \gamma + \sin^2 \beta \sin^2 \gamma}{1 - \sin^2 \beta \sin^2 \gamma}$$

$$\Rightarrow \tan^2 \alpha = \frac{\sin^2 \beta - \sin^2 \beta \sin^2 \gamma + \sin^2 \gamma - \sin^2 \beta \sin^2 \gamma}{\cos^2 \beta - \sin^2 \gamma (1 - \sin^2 \beta)}$$

$$= \frac{\sin^2 \beta \cos^2 \gamma + \cos^2 \beta \sin^2 \gamma}{\cos^2 \beta \cos^2 \gamma}$$

$$= \tan^2 \beta + \tan^2 \gamma$$

$$\Rightarrow \tan^2 \alpha - \tan^2 \beta - \tan^2 \gamma = 0$$

20. (a) Given $\tan A/2, \tan B/2, \tan C/2$ are in A.P.

$$\therefore \tan A/2 - \tan B/2 = \tan B/2 - \tan C/2$$

$$\therefore \frac{\sin A/2}{\cos A/2} - \frac{\sin B/2}{\cos B/2} = \frac{\sin B/2}{\cos B/2} - \frac{\sin C/2}{\cos C/2}$$

$$\Rightarrow \frac{\sin A/2 \cdot \cos B/2 - \sin B/2 \cdot \cos A/2}{\cos A/2 \cdot \cos B/2}$$

$$= \frac{\sin B/2 \cdot \cos C/2 - \sin C/2 \cdot \cos B/2}{\cos B/2 \cdot \cos C/2}$$

$$\Rightarrow \frac{\sin\left(\frac{A-B}{2}\right)}{\cos A/2} = \frac{\sin\left(\frac{B-C}{2}\right)}{\cos C/2}$$

(1)

$$\text{But } \cos A/2 = \sin \frac{B+C}{2} \text{ and } \cos C/2 = \sin \frac{A+B}{2} \quad (2)$$

$$\Rightarrow \sin\left(\frac{A-B}{2}\right) \cdot \sin\left(\frac{A+B}{2}\right) = \sin\left(\frac{B-C}{2}\right)$$

$$\sin\left(\frac{B+C}{2}\right) \text{ \{from (1) and (2)\}}$$

$$\Rightarrow \cos B - \cos A = \cos C - \cos B$$

Hence, $\cos A, \cos B, \cos C$ are in A.P.

DPP 3.2

Single Correct Answer Type

$$1. (c) \frac{\sin(\pi - \alpha)}{\sin \alpha - \cos \alpha \tan \frac{\alpha}{2}} - \cos \alpha$$

$$= \frac{\sin \alpha}{\sin \alpha - \cos \alpha \frac{\sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}}} - \cos \alpha$$

$$= \frac{\sin \alpha \cos \frac{\alpha}{2}}{\sin \alpha \cos \frac{\alpha}{2} - \cos \alpha \sin \frac{\alpha}{2}} - \cos \alpha$$

$$= \frac{2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \cos \frac{\alpha}{2}}{\sin\left(\alpha - \frac{\alpha}{2}\right)} - \cos \alpha$$

$$= 2 \cos^2 \frac{\alpha}{2} - \cos \alpha$$

$$= 1$$

$$2. (d) \frac{1 + \sin 2\alpha}{\cos 2\alpha \cdot \tan\left(\frac{\pi}{4} + \alpha\right)} - \frac{\sin 2\alpha}{4} \left[\cot \frac{\alpha}{2} - \tan \frac{\alpha}{2} \right]$$

$$= \frac{1 + \sin 2\alpha}{\cos 2\alpha \cdot \tan\left(\frac{\pi}{4} + \alpha\right)} - \frac{\sin 2\alpha}{4} \left[\frac{\cos^2 \frac{\alpha}{2} - \sin^2 \frac{\alpha}{2}}{\cos \frac{\alpha}{2} \sin \frac{\alpha}{2}} \right]$$

$$= \frac{(\sin \alpha + \cos \alpha)^2}{\cos 2\alpha (\sin \alpha + \cos \alpha)} - \cos^2 \alpha$$

$$= \frac{\cos 2\alpha}{\cos 2\alpha} - \cos^2 \alpha$$

$$= \sin^2 \alpha$$

$$3. (c) \tan^2 \frac{\theta}{2} = \frac{1 - \cos \theta}{1 + \cos \theta} = \frac{\tan \alpha - \tan \beta}{\tan \alpha + \tan \beta} = \frac{\sin(\alpha - \beta)}{\sin(\alpha + \beta)}$$

$$4. (b) \operatorname{cosec} \theta = \frac{p+q}{p-q} \Rightarrow \frac{1}{\sin \theta} = \frac{p+q}{p-q}$$

Apply componendo and dividendo

$$\frac{1 + \sin \theta}{1 - \sin \theta} = \frac{p+q+p-q}{p+q-p+q}$$

$$\Rightarrow \left\{ \frac{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}} \right\}^2 = \frac{p}{q}$$

$$\Rightarrow \left\{ \frac{1 + \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2}} \right\}^2 = \frac{p}{q}$$

$$\Rightarrow \tan^2 \left(\frac{\pi}{4} + \frac{\theta}{2} \right) = \frac{p}{q}$$

$$\Rightarrow \cot^2 \left(\frac{\pi}{4} + \frac{\theta}{2} \right) = \frac{q}{p}$$

$$5. (c) \sin(x + 3\alpha) = 3 \sin(\alpha - x)$$

$$\Rightarrow \sin x \cos 3\alpha + \sin 3\alpha \cos x = 3(\sin \alpha \cos x - \sin x \cos \alpha)$$

$$\Rightarrow \sin x(\cos 3\alpha + 3 \cos \alpha) = \cos x(3 \sin \alpha - \sin 3\alpha)$$

$$\Rightarrow \tan x = \frac{4 \sin^3 \alpha}{4 \cos^3 \alpha} = \tan^3 \alpha$$

$$6. (b) 2(\cos^2 20^\circ + \cos^2 140^\circ + \cos^2 100^\circ)$$

$$= 3 + \cos 40^\circ + \cos 280^\circ + \cos 200^\circ$$

$$= 3 + 2 \cos 120^\circ \cos 160^\circ + \cos 200^\circ$$

$$= 3 - \cos 160^\circ + \cos 200^\circ$$

$$= 3 - \cos(360^\circ - 200^\circ) + \cos 200^\circ$$

$$= 3 - \cos 200^\circ + \cos 200^\circ$$

$$= 3$$

$$7. (b) \text{ Given, } \sin 2\theta + \sin 2\phi = 1/2 \quad (1)$$

$$\text{and } \cos 2\theta + \cos 2\phi = 3/2 \quad (2)$$

Square and adding,

$$\therefore (\sin^2 2\theta + \cos^2 2\theta) + (\sin^2 2\phi + \cos^2 2\phi)$$

$$+ 2[\sin 2\theta \sin 2\phi + \cos 2\theta \cos 2\phi] = 1/4 + 9/4$$

$$\Rightarrow \cos 2\theta \cos 2\phi + \sin 2\theta \sin 2\phi = 1/4$$

$$\Rightarrow \cos(2\theta - 2\phi) = 1/4 \Rightarrow \cos^2(\theta - \phi) = 5/8$$

$$8. (d) f(x) = 1 - \cos^2(x + \alpha) + \sin^2(x + \beta) - 2 \cos(\alpha - \beta) \sin(x + \alpha) \sin(x + \beta)$$

$$= 1 - \cos(2x + \alpha + \beta) \cos(\alpha - \beta)$$

$$- 2 \cos(\alpha - \beta) \sin(x + \alpha) \sin(x + \beta)$$

$$= 1 - \cos(\alpha - \beta) [\cos(2x + \alpha + \beta) - \cos(2x + \alpha + \beta) + \cos(\alpha - \beta)]$$

$$= \sin^2(\alpha - \beta)$$

which is a constant function.

$$9. (d) \text{ From the given relation, we have}$$

$$2 \cos^2 B - 1 = 3(1 - \cos^2 A)$$

$$\text{or } \cos 2B = 3 \sin^2 A$$

$$(1)$$

$$\frac{3 \sin A}{\sin B} = \frac{2 \cos B}{\cos A}$$

$$\Rightarrow \frac{3}{2} \sin 2A = \sin 2B$$

$$\Rightarrow 3 \sin 2A = 2 \sin 2B$$

Now $\cos(A + 2B)$

$$= \cos A \cos 2B - \sin A \sin 2B$$

$$= \cos A (3 \sin^2 A) - \sin A \cdot \frac{3}{2} \sin 2A$$

$$= 3 \cos A \sin^2 A - 3 \sin^2 A \cos A = 0$$

$$\therefore A + 2B = 90^\circ$$

$$\begin{aligned} 10. (c) \quad & \frac{1}{2 \cos 40^\circ} + 4 \cos 40^\circ \cdot \cos 20^\circ - \frac{1}{2 \cos 20^\circ} \\ &= \frac{1}{2} \left[\frac{1}{\cos 40^\circ} - \frac{1}{\cos 20^\circ} \right] + 2(\cos 60^\circ + \cos 20^\circ) \\ &= \frac{1}{2} \left[\frac{\cos 20^\circ - \cos 40^\circ}{\cos 40^\circ \cos 20^\circ} \right] + 1 + 2 \cos 20^\circ \\ &= \frac{2 \sin 30^\circ \sin 10^\circ}{2 \cos 40^\circ \cos 20^\circ} + 1 + 2 \cos 20^\circ \\ &= \frac{2 \sin 10^\circ \sin 20^\circ}{\sin 80^\circ} + 1 + 2 \cos 20^\circ \\ &= \frac{2 \sin 10^\circ \sin 20^\circ + 2 \cos 20^\circ \cos 10^\circ}{\sin 80^\circ} + 1 \\ &= \frac{\cos 10^\circ - \cos 30^\circ + \cos 30^\circ + \cos 10^\circ}{\sin 80^\circ} + 1 \\ &= \frac{2 \cos 10^\circ}{\sin 80^\circ} + 1 = 3 \end{aligned}$$

$$11. (d) \text{ Let } A = \alpha, B = 2\alpha, C = 4\alpha$$

$$\therefore A + B + C = 7\alpha = 2\pi$$

$$\therefore \cos(A + B + C) = \cos 2\pi = 1$$

$$\therefore \cos A \cdot \cos B \cdot \cos C - \sin A \cdot \sin B \cdot \cos C - \sin B \cdot \sin C \cdot \cos A - \sin C \cdot \sin A \cdot \cos B = 1$$

Dividing throughout by $\cos A \cdot \cos B \cdot \cos C$, we get

$$1 - (\tan A \tan B + \tan B \tan C + \tan C \tan A)$$

$$= \frac{1}{\cos A \cdot \cos B \cdot \cos C}$$

$$\therefore \tan A \tan B + \tan B \tan C + \tan C \tan A$$

$$= 1 - \frac{1}{\cos \alpha \cdot \cos 2\alpha \cdot \cos 4\alpha}$$

$$= 1 - \frac{1}{\frac{\sin 2^3 \alpha}{2^3 \sin \alpha}}$$

$$= 1 - \frac{8 \sin \alpha}{\sin 8\alpha} = 1 - \frac{8 \sin \frac{2\pi}{7}}{\sin \frac{16\pi}{7}}$$

$$= 1 - 8 = -7$$

$$\left[\because \sin \frac{16\pi}{7} = \sin \left(2\pi + \frac{2\pi}{7} \right) = \sin \frac{2\pi}{7} \right]$$

$$12. (c) \quad x = \sqrt{2 + \sqrt{2 - \sqrt{2 + x}}}$$

We must have $x = 2 - \sqrt{2 + x} \geq 0$

(2)

$$\Rightarrow 2 \geq \sqrt{2 + x}$$

$$\Rightarrow x \leq 2$$

$$\text{Also } 2 + x \geq 0$$

$$\Rightarrow x \geq -2$$

$$\therefore x \in [-2, 2]$$

Let $x = 2 \cos \theta$, where $\theta \in [0, \pi]$. Then,

$$x = \sqrt{2 + \sqrt{2 - \sqrt{2 + 2 \cos \theta}}}$$

$$\Rightarrow 2 \cos \theta = \sqrt{2 + \sqrt{2 - 2 \cos \frac{\theta}{2}}}$$

$$= \sqrt{2 + \sqrt{2 \left(1 - \cos \frac{\theta}{2} \right)}}$$

$$= \sqrt{2 + 2 \sin \frac{\theta}{4}}$$

$$= \sqrt{2 + 2 \cos \left(\frac{\pi}{2} - \frac{\theta}{4} \right)}$$

$$= \sqrt{2 \left(1 + \cos \left(\frac{\pi}{2} - \frac{\theta}{4} \right) \right)}$$

$$\Rightarrow 2 \cos \theta = 2 \cos \left(\frac{\pi}{4} - \frac{\theta}{8} \right)$$

$$\Rightarrow \theta = \frac{\pi}{4} - \frac{\theta}{8}$$

$$\Rightarrow \theta = \frac{2\pi}{9}$$

$$\text{Hence, } x = 2 \cos \frac{2\pi}{9} = 2 \cos 40^\circ$$

$$13. (a) \quad 3 \cos 2\phi - 3 \cos 2\theta = 1 - \cos 2\phi \cos 2\theta$$

$$\Rightarrow (3 + \cos 2\theta) \cos 2\phi = 1 + 3 \cos 2\theta$$

$$\Rightarrow \cos 2\phi = \frac{1 + 3 \cos 2\theta}{3 + \cos 2\theta}$$

$$\Rightarrow \frac{1 - \cos 2\phi}{1 + \cos 2\phi} = \frac{1}{2} \cdot \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

(using componendo and dividendo)

$$\Rightarrow \tan^2 \phi = \frac{1}{2} \tan^2 \theta$$

$$\Rightarrow \tan \theta = \sqrt{2} \tan \phi$$

$$\Rightarrow k = \sqrt{2}$$

$$14. (a) \quad f(\theta) = \sin^2 \theta + 3 \sin \theta \cos \theta + 5 \cos^2 \theta$$

$$= 1 + 4 \cos^2 \theta + \frac{3}{2} \sin 2\theta$$

$$= 3 + 2 \cos 2\theta + \frac{3}{2} \sin 2\theta$$

Minimum of $f(\theta)$ and $f\left(\frac{\pi}{2} - \theta\right)$ will be same because of the

type of function $a \sin \theta + b \cos \theta$ and is $3 - \frac{5}{2} = \frac{1}{2}$

$$\therefore g(\theta) = \frac{1}{2}$$

$$\therefore \frac{1}{g(\theta)} = 2$$

S.16 Trigonometry

$$15. (c) \quad 3 \sin^2 x + 3 \sin x \cdot \cos x + 7 \cos^2 x$$

$$= \frac{3(1 - \cos 2x)}{2} + \frac{3 \sin 2x}{2} + \frac{7(1 + \cos 2x)}{2}$$

$$= \frac{3 \sin 2x + 4 \cos 2x}{2} + 5$$

$$\text{Now } -5 \leq 3 \sin 2x + 4 \cos 2x \leq 5$$

$$\therefore \frac{3 \sin 2x + 4 \cos 2x}{2} + 5 \in \left[\frac{5}{2}, \frac{15}{2} \right]$$

$$16. (c) \quad S = \sum_{r=1}^{11} \tan^2 \left(\frac{r\pi}{24} \right)$$

$$= \left(\tan^2 \frac{\pi}{24} + \tan^2 \frac{11\pi}{24} \right) + \left(\tan^2 \frac{2\pi}{24} + \tan^2 \frac{10\pi}{24} \right)$$

$$+ \left(\tan^2 \frac{3\pi}{24} + \tan^2 \frac{9\pi}{24} \right) + \left(\tan^2 \frac{4\pi}{24} + \tan^2 \frac{8\pi}{24} \right)$$

$$+ \left(\tan^2 \frac{5\pi}{24} + \tan^2 \frac{7\pi}{24} \right) + \left(\tan^2 \frac{6\pi}{24} \right)$$

$$= \left(\tan^2 \frac{\pi}{24} + \cot^2 \frac{\pi}{24} \right)$$

$$+ \left\{ (2 - \sqrt{3})^2 + (2 + \sqrt{3})^2 + (\sqrt{2} - 1)^2 + (\sqrt{2} + 1)^2 \right\}$$

$$+ \left\{ \left(\frac{1}{\sqrt{3}} \right)^2 + (\sqrt{3})^2 \right\}$$

$$+ \left(\tan^2 \frac{5\pi}{24} + \cot^2 \frac{5\pi}{24} \right) + 1$$

$$\text{Now } (\tan^2 \theta + \cot^2 \theta) = 2 + 4 \cot^2 2\theta$$

$$S = 2 + 4 \cot^2 \frac{\pi}{12} + \frac{70}{3} + 2 + 4 \cot^2 \frac{5\pi}{12} + 1$$

$$= \frac{85}{3} + 4[(2 + \sqrt{3})^2 + (2 - \sqrt{3})^2]$$

$$= \frac{85}{3} + 4(2)(4 + 3) = \frac{253}{3}$$

$\therefore$

$$17. (c) \quad \tan \theta = \frac{3 \tan \frac{\theta}{3} - \tan^3 \frac{\theta}{3}}{1 - 3 \tan^2 \frac{\theta}{3}} = \lambda$$

$$\Rightarrow \tan^3 \left(\frac{\theta}{3} \right) - 3\lambda \tan^2 \left(\frac{\theta}{3} \right) - 3 \tan \frac{\theta}{3} + \lambda = 0$$

$$\text{Above equation has roots } \tan \frac{\theta_1}{3}, \tan \frac{\theta_2}{3}, \tan \frac{\theta_3}{3}$$

$$\therefore \left| \sum \tan \left(\frac{\theta_i}{3} \right) \right| = |-3| = 3$$

$$18. (a) \quad u = 1 + \cos \theta + \cos 2\theta + \cos (\theta + 2\theta)$$

$$= (1 + \cos 3\theta) + (\cos \theta + \cos 2\theta)$$

$$= 2 \cos^2 \frac{3\theta}{2} + 2 \cos \frac{3\theta}{2} \cos \frac{\theta}{2}$$

$$= 2 \cos \frac{3\theta}{2} \left[\cos \frac{3\theta}{2} + \cos \frac{\theta}{2} \right]$$

$$\text{Similarly } v = 2 \sin \frac{3\theta}{2} \left[\cos \frac{3\theta}{2} + \cos \frac{\theta}{2} \right]$$

$$\therefore u^2 + v^2 = 4 \left(\cos \frac{3\theta}{2} + \cos \frac{\theta}{2} \right)^2$$

$$= 4 \left(2 \cos \theta \cdot \cos \frac{\theta}{2} \right)^2 = 16 \cos^2 \theta \cdot \cos^2 \frac{\theta}{2}$$

$$= 4(1 + \cos \theta)(1 + \cos 2\theta)$$

Multiple Correct Answers Type

19. (b, c)

$$\sec x + \tan x = \frac{22}{7}$$

$$\Rightarrow \frac{1 + \sin x}{\cos x} = \frac{22}{7}$$

$$\Rightarrow \frac{1 + \frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2}} = \frac{22}{7}, \text{ where } t = \tan \frac{x}{2}$$

$$\Rightarrow \frac{(1+t)^2}{1-t^2} = \frac{22}{7}$$

$$\Rightarrow \frac{1+t}{1-t} = \frac{22}{7}$$

$$\Rightarrow t = \frac{15}{29}$$

$$\text{Now } \operatorname{cosec} x + \cot x$$

$$= \frac{1 + \cos x}{\sin x}$$

$$= \frac{2 \cos^2(x/2)}{2 \sin(x/2) \cos(x/2)}$$

$$= \cot \left(\frac{x}{2} \right) = \frac{29}{15}$$

20. (a, b, c, d)

$$\text{Since } 1 \pm \sin \theta = \left(\cos \frac{\theta}{2} \pm \sin \frac{\theta}{2} \right)^2$$

$$\therefore y = \frac{\pm(\cos 2x - \sin 2x) + 1}{\pm(\cos 2x + \sin 2x) - 1}$$

Taking '+' sign in Nr. and Dr. we get

$$y = \frac{1 + \cos 2x - \sin 2x}{\sin 2x - (1 - \cos 2x)}$$

$$= \frac{2 \cos^2 x - 2 \sin x \cos x}{2 \sin x \cos x - 2 \sin^2 x} = \frac{\cos x}{\sin x} = \cot x$$

Taking '-' sign in Nr. and Dr. we get

$$y = \frac{\sin 2x + 1 - \cos 2x}{-\sin 2x - (1 + \cos 2x)}$$

$$= \frac{2 \sin x \cos x + 2 \sin^2 x}{-2 \sin x \cos x - 2 \cos^2 x}$$

$$= -\tan x$$

Taking '+' sign in Nr. and '-' sign in Dr. we get

$$\begin{aligned}
 y &= \frac{1 + \cos 2x - \sin 2x}{-\sin 2x - (1 + \cos 2x)} \\
 &= \frac{2 \cos^2 x - 2 \sin x \cos x}{-2 \sin x \cos x - 2 \cos^2 x} \\
 &= \frac{\sin x - \cos x}{\sin x + \cos x} \\
 &= \tan \left(x - \frac{\pi}{4} \right) \\
 &= -\cot \left(\frac{\pi}{2} + x - \frac{\pi}{4} \right) \\
 &= -\cot \left(\frac{\pi}{4} + x \right)
 \end{aligned}$$

Taking '-' sign in Nr. and '+' sign in Dr. we get

$$\begin{aligned}
 y &= \frac{\sin 2x + 1 - \cos 2x}{\sin 2x - (1 - \cos 2x)} \\
 &= \frac{2 \sin x \cos x + 2 \sin^2 x}{2 \sin x \cos x - 2 \sin^2 x} \\
 &= \frac{\cos x + \sin x}{\cos x - \sin x} \\
 &= \frac{1 + \tan x}{1 - \tan x} \\
 &= \tan \left(\frac{\pi}{4} + x \right)
 \end{aligned}$$

21. (b, c, d)

$$(a-b) \sin(\theta + \phi) = (a+b) \sin(\theta - \phi)$$

$$\Rightarrow \frac{\sin(\theta + \phi)}{\sin(\theta - \phi)} = \frac{a+b}{a-b}$$

$$\Rightarrow \frac{\sin(\theta + \phi) + \sin(\theta - \phi)}{\sin(\theta + \phi) - \sin(\theta - \phi)} = \frac{2a}{2b}$$

$$\Rightarrow \frac{2 \sin \theta \cos \phi}{2 \cos \theta \sin \phi} = \frac{a}{b} = \frac{\tan \theta}{\tan \phi} = \frac{a}{b}$$

$$\Rightarrow b \tan \theta = a \tan \phi$$

$\therefore$ (b) is true

$$\Rightarrow \frac{2a \tan \frac{\phi}{2}}{1 - \tan^2 \frac{\phi}{2}} = \frac{2b \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$$

$$\Rightarrow \frac{a \left(\frac{a \tan \frac{\theta}{2} - c}{b} \right)}{1 - \left(\frac{a \tan \frac{\theta}{2} - c}{b} \right)^2} = \frac{b \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$$

$$\begin{aligned}
 &\Rightarrow \frac{\left(a^2 \tan \frac{\theta}{2} - ac \right)}{b^2 - \left(a^2 \tan^2 \frac{\theta}{2} + c^2 - 2ac \tan \frac{\theta}{2} \right)} \\
 &= \frac{\tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}
 \end{aligned}$$

$$\begin{aligned}
 &\Rightarrow \left(a^2 \tan \frac{\theta}{2} - ac \right) \left(1 - \tan^2 \frac{\theta}{2} \right) \\
 &= b^2 \tan \frac{\theta}{2} - a^2 \tan^3 \frac{\theta}{2} - c^2 \tan \frac{\theta}{2} + 2ac \tan^2 \frac{\theta}{2} \\
 &= a^2 \tan \frac{\theta}{2} - ac - a^2 \tan^3 \frac{\theta}{2} + ac \tan^2 \frac{\theta}{2} \\
 &= b^2 \tan \frac{\theta}{2} - a^2 \tan^3 \frac{\theta}{2} - c^2 \tan \frac{\theta}{2} + 2ac \tan^2 \frac{\theta}{2}
 \end{aligned}$$

$$\Rightarrow (a^2 + c^2 - b^2) \tan \frac{\theta}{2} = ac \left[1 + \tan^2 \frac{\theta}{2} \right]$$

$$\Rightarrow \frac{2 \tan \frac{\theta}{2}}{1 + \tan^2 \frac{\theta}{2}} = \frac{2ac}{c^2 + a^2 - b^2}$$

$$\Rightarrow \sin \theta = \frac{2ac}{c^2 + a^2 - b^2}$$

$$\text{Similarly, } \sin \phi = \frac{2bc}{a^2 - b^2 - c^2}$$

22. (a, c, d)

From the given results

$$\begin{aligned}
 &\cos x \sin^2 \beta \cos \alpha - \cos^2 \alpha \sin^2 \beta \\
 &= \sin^2 \alpha \cos \beta \cos x - \sin^2 \alpha \cos^2 \beta
 \end{aligned}$$

$$\begin{aligned}
 &\Rightarrow \cos x [\cos \alpha \sin^2 \beta - \sin^2 \alpha \cos \beta] \\
 &= \cos^2 \alpha \sin^2 \beta - \sin^2 \alpha \cos^2 \beta
 \end{aligned}$$

$$\Rightarrow \cos x = \frac{\cos^2 \alpha [1 - \cos^2 \beta] - (1 - \cos^2 \alpha) \cos^2 \beta}{\cos \alpha [1 - \cos^2 \beta] - [1 - \cos^2 \alpha] \cos \beta}$$

$$= \frac{\cos^2 \alpha - \cos^2 \beta}{(\cos \alpha - \cos \beta)(\cos \alpha \cos \beta + 1)}$$

$$= \frac{\cos \alpha + \cos \beta}{1 + \cos \alpha \cos \beta}$$

$$\Rightarrow \frac{\cos x}{1} = \frac{\cos \alpha + \cos \beta}{1 + \cos \alpha \cos \beta}$$

$$\Rightarrow \frac{1 - \cos x}{1 + \cos x} = \frac{1 + \cos \alpha \cos \beta - \cos \alpha - \cos \beta}{1 + \cos \alpha \cos \beta + \cos \alpha + \cos \beta}$$

$$= \frac{(1 - \cos \alpha)(1 - \cos \beta)}{(1 + \cos \alpha)(1 + \cos \beta)}$$

$$\Rightarrow \tan^2 \frac{x}{2} = \tan^2 \frac{\alpha}{2} \cdot \tan^2 \frac{\beta}{2}$$

$$\Rightarrow \tan \frac{x}{2} = \pm \tan \frac{\alpha}{2} \tan \frac{\beta}{2}$$

23. (b, d)

$$\sin \frac{\theta}{2} = \sqrt{1 + \sin \theta} - \sqrt{1 - \sin \theta}$$

$$\Rightarrow \sin \frac{\theta}{2} = \left| \cos \frac{\theta}{2} + \sin \frac{\theta}{2} \right| - \left| \cos \frac{\theta}{2} - \sin \frac{\theta}{2} \right|$$

$$\text{If } 0 \leq \theta \leq \frac{\pi}{2}, \sin \frac{\theta}{2} = 2 \sin \frac{\theta}{2} \Rightarrow \theta = 0$$

$$\text{If } \frac{\pi}{2} < \theta \leq \pi, \sin \frac{\theta}{2} = 2 \cos \frac{\theta}{2}$$

$$\Rightarrow \tan \frac{\theta}{2} = 2$$

$$\Rightarrow \tan \theta = \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}} = \frac{4}{1 - 4} = \frac{-4}{3}$$

24. (a, b, c)

$$(\alpha \tan \gamma + \beta \cot \gamma)(\alpha \cot \gamma + \beta \tan \gamma) - 4\alpha\beta \cot^2 2\gamma$$

$$= \alpha^2 + (\tan^2 \gamma + \cot^2 \gamma) \alpha\beta + \beta^2 - 4\alpha\beta \frac{\cos^2 2\gamma}{\sin^2 2\gamma}$$

$$= \alpha^2 + \beta^2 + \alpha\beta \left[\frac{\sin^2 \gamma}{\cos^2 \gamma} + \frac{\cos^2 \gamma}{\sin^2 \gamma} \right] - \frac{4(\cos^2 \gamma - \sin^2 \gamma)^2}{4 \sin^2 \gamma \cos^2 \gamma}$$

$$= \alpha^2 + \beta^2 + \alpha\beta \left[\frac{(\sin^4 \gamma + \cos^4 \gamma) - (\cos^4 \gamma + \sin^4 \gamma - 2 \sin^2 \gamma \cos^2 \gamma)}{\sin^2 \gamma \cos^2 \gamma} \right]$$

$$= \alpha^2 + \beta^2 + 2\alpha\beta = (\alpha + \beta)^2, \text{ which is independent on } \gamma \text{ and dependent on } \alpha, \beta.$$

DPP 3.3

Single Correct Answer Type

$$\begin{aligned} 1. (a) \quad & 4 \cos 18^\circ - \frac{3}{\cos 18^\circ} - 2 \tan 18^\circ \\ &= \frac{4 \cos^2 18^\circ - 3 - 2 \sin 18^\circ}{\cos 18^\circ} \\ &= \frac{2(1 + \cos 36^\circ) - 2 \sin 18^\circ - 3}{\cos 18^\circ} \\ &= \frac{2(1 + \cos 36^\circ - \sin 18^\circ) - 3}{\cos 18^\circ} \\ &= \frac{2(1 + \cos 36^\circ - \sin 18^\circ) - 3}{\cos 18^\circ} = 0 \end{aligned}$$

$$\begin{aligned} 2. (a) \quad & \frac{\cot 84^\circ \cot 48^\circ}{\cot 66^\circ \cot 78^\circ} \\ &= \frac{(2 \cos 84^\circ \sin 66^\circ)(2 \sin 78^\circ \cos 48^\circ)}{(2 \sin 84^\circ \cos 66^\circ)(2 \cos 78^\circ \sin 48^\circ)} \\ &= \frac{(\sin 150^\circ - \sin 18^\circ)(\sin 126^\circ + \sin 30^\circ)}{(\sin 150^\circ + \sin 18^\circ)(\sin 126^\circ - \sin 30^\circ)} \\ &= \frac{(\sin 30^\circ - \sin 18^\circ)(\sin 54^\circ + \sin 30^\circ)}{(\sin 30^\circ + \sin 18^\circ)(\sin 54^\circ - \sin 30^\circ)} \\ &= \frac{\left(\frac{1}{2} - \frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4} + \frac{1}{2}\right)}{\left(\frac{1}{2} + \frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4} - \frac{1}{2}\right)} \\ &= \frac{(3-\sqrt{5})(3+\sqrt{5})}{(\sqrt{5}+1)(\sqrt{5}-1)} = \frac{9-5}{5-1} = \frac{4}{4} = 1 \end{aligned}$$

$$3. (c) \quad \text{Using } \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\begin{aligned} \tan 7.5^\circ &= \frac{\sin 15^\circ}{1 + \cos 15^\circ} \\ &= \frac{\sin 15^\circ}{1 + \cos 15^\circ} \times \frac{1 - \cos 15^\circ}{1 - \cos 15^\circ} \\ &= \frac{\sin 15^\circ(1 - \cos 15^\circ)}{\sin^2 15^\circ} \\ &= \frac{1 - \cos 15^\circ}{\sin 15^\circ} \\ &= \frac{1 - \frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} \\ &= \frac{2\sqrt{2} - \sqrt{3} - 1}{\sqrt{3} - 1} \\ &= \frac{2\sqrt{2} - \sqrt{3} - 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \\ &= \frac{(2\sqrt{2} - \sqrt{3} - 1)(\sqrt{3} + 1)}{3 - 1} \\ &= \frac{2\sqrt{6} + 2\sqrt{2} - 4 - 2\sqrt{3}}{2} \\ &= \frac{2(\sqrt{6} + \sqrt{2} - 2 - \sqrt{3})}{2} \\ &= \sqrt{3}(\sqrt{2} - 1) - \sqrt{2}(\sqrt{2} - 1) \\ &= (\sqrt{3} - \sqrt{2})(\sqrt{2} - 1) \end{aligned}$$

4. (a) Given

$$\sin(A+B) + \sin(A-B) + \sin(B+C) + \sin(B-C) + \sin(C+A) + \sin(C-A)$$

$$= \sin A + \sin B + \sin C$$

$$\Rightarrow \sin(A-B) + \sin(B-C) + \sin(C-A) = 0$$

$$\Rightarrow 4 \sin \frac{A-B}{2} \sin \frac{B-C}{2} \sin \frac{C-A}{2} = 0$$

$\therefore \Delta$ is isosceles

5. (c) $\cos A, \cos B$ and $\cos C$ are the roots of the cubic equation

$$x^3 + ax^2 + bx + c = 0$$

$$\Rightarrow \cos A + \cos B + \cos C = -a$$

$$\cos A \cos B + \cos B \cos C + \cos C \cos A = b$$

$$\cos A \cos B \cos C = -c$$

$$\text{Now } (\cos A + \cos B \cos C)^2 = (\Sigma \cos^2 A) + 2(\Sigma \cos A \cos B)$$

$$\therefore \cos^2 A + \cos^2 B + \cos^2 C = a^2 - 2b$$

$$\therefore 3 + \cos 2A + \cos 2B + \cos 2C = 2a^2 - 4b$$

$$\therefore 3 - 1 - 4 \cos A \cos B \cos C = 2a^2 - 4b$$

$$\therefore 1 + 2c = a^2 - 2b$$

$$\therefore a^2 - 2b - 2c = 1$$

6. (a) L.H.S. = $(\cos A + \cos B) + (\cos C + \cos D)$

$$\begin{aligned}
 &= 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} + 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2} \\
 &= 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} + 2 \cos \left(\pi - \frac{A+B}{2} \right) \cos \frac{C-D}{2} \\
 &= 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} - 2 \cos \frac{A+B}{2} \cos \frac{C-D}{2} \\
 &= 2 \cos \frac{A+B}{2} \left[\cos \frac{A-B}{2} - \cos \frac{C-D}{2} \right] \\
 &= 2 \cos \frac{A+B}{2} \times 2 \sin \frac{A-B+C-D}{4} \sin \frac{C-D-A+B}{4} \\
 &= 4 \cos \frac{A+B}{2} \sin \frac{A+C-(B+D)}{4} \sin \frac{B+C-(A+D)}{4} \\
 &= 4 \cos \frac{A+B}{2} \sin \frac{A+C-(2\pi-A-C)}{4} \sin \frac{B+C-(2\pi-B-C)}{4} \\
 &= 4 \cos \frac{A+B}{2} \sin \left(\frac{A+C}{2} - \frac{\pi}{2} \right) \sin \left(\frac{B+C}{2} - \frac{\pi}{2} \right) \\
 &= 4 \cos \frac{A+B}{2} \left[-\sin \left(\frac{\pi}{2} - \frac{A+C}{2} \right) \right] \left[-\sin \left(\frac{\pi}{2} - \frac{B+C}{2} \right) \right] \\
 &= 4 \cos \frac{A+B}{2} \cos \frac{A+C}{2} \cos \frac{B+C}{2}
 \end{aligned}$$

7. (b) $A + B + C + D = 2\pi$

$$\therefore A + B = 2\pi - (C + D)$$

$$\therefore \tan(A + B) = \tan[2\pi - (C + D)] = -\tan(C + D)$$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = -\frac{\tan C + \tan D}{1 - \tan C \tan D}$$

$$\Rightarrow (\tan A + \tan B)(1 - \tan C \tan D)$$

$$= -(1 - \tan A \tan B)(\tan C + \tan D)$$

$$\Rightarrow \tan A + \tan B - \tan A \tan C \tan D - \tan B \tan C \tan D$$

$$= -[(\tan C + \tan D - \tan A \tan B \tan C - \tan A \tan B \tan D)]$$

$$\Rightarrow \tan A + \tan B + \tan C + \tan D$$

$$= \tan A \tan B \tan C + \tan A \tan B \tan D$$

$$+ \tan A \tan B \tan D + \tan B \tan C \tan D$$

Dividing both sides by $\tan A \tan B \tan C \tan D$, we get

$$\frac{\tan A + \tan B + \tan C + \tan D}{\tan A \tan B \tan C \tan D}$$

$$= \frac{1}{\tan D} + \frac{1}{\tan B} + \frac{1}{\tan C} + \frac{1}{\tan A}$$

$$= \cot D + \cot B + \cot C + \cot A$$

$$\Rightarrow \frac{\tan A + \tan B + \tan C + \tan D}{\cot A + \cot B + \cot C + \cot D} = \tan A \tan B \tan C \tan D$$

8. (b) $\cos^2 \frac{\pi}{11} + \cos^2 \frac{2\pi}{11} + \cos^2 \frac{3\pi}{11} + \cos^2 \frac{4\pi}{11} + \cos^2 \frac{5\pi}{11}$

$$= \frac{1}{2} \left[\left(1 + \cos \frac{2\pi}{11} \right) + \left(1 + \cos \frac{4\pi}{11} \right) + \dots + \left(1 + \cos \frac{10\pi}{11} \right) \right]$$

$$= \frac{1}{2} \left[5 + \left(\cos \frac{2\pi}{11} + \cos \frac{4\pi}{11} + \cos \frac{6\pi}{11} + \dots + \cos \frac{10\pi}{11} \right) \right]$$

$$= \frac{1}{2} \left[5 + \frac{\sin \frac{5\pi}{11}}{\sin \frac{\pi}{11}} \cos \left(\frac{2\pi}{11} + \frac{10\pi}{11} \right) \right]$$

$$= \frac{1}{2} \left[5 + \frac{\sin \frac{5\pi}{11}}{\sin \frac{\pi}{11}} \cos \frac{6\pi}{11} \right]$$

$$= \frac{1}{2} \left[5 - \frac{2 \sin \frac{5\pi}{11} \cos \frac{5\pi}{11}}{2 \sin \frac{\pi}{11}} \right]$$

$$= \frac{1}{2} \left[5 - \frac{\sin \frac{10\pi}{11}}{2 \sin \frac{\pi}{11}} \right]$$

$$= \frac{1}{2} \left[5 - \frac{1}{2} \right]$$

$$= \frac{9}{4}$$

9. (a) $\cos(-89^\circ) + \cos(-87^\circ) + \cos(-85^\circ) + \dots + \cos(85^\circ) + \cos(87^\circ) + \cos(89^\circ)$

$$= 2(\cos 1^\circ + \cos 3^\circ + \cos 5^\circ + \dots + \cos 89^\circ)$$

$$= 2 \cdot \frac{\sin(45^\circ)}{\sin(1^\circ)} \left\{ \cos \left(\frac{1^\circ + 89^\circ}{2} \right) \right\}$$

$$= \frac{2 \sin 45^\circ \cos 45^\circ}{\sin 1^\circ}$$

$$= \frac{\sin 90^\circ}{\sin 1^\circ}$$

$$= \operatorname{cosec} 1^\circ$$

10. (d) In a $\triangle ABC$, $\cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2} \geq 3\sqrt{3}$

So there is no triangle possible with $\cot \frac{A}{2} \cot \frac{B}{2} \cot \frac{C}{2} = 4$.

11. (a) Let $x_i = \sin \theta_i$ [$i = 1, 2, \dots, 10$]

$$\text{Given } \sum_{i=1}^{10} (\sin \theta_i)^3 = 0$$

$$\Rightarrow 4 \sum_{k=1}^{10} (\sin \theta_i)^3 = 0$$

$$\Rightarrow 3 \sum_{i=1}^{10} \sin \theta_i - \sum_{i=1}^{10} \sin 3\theta_i = 0 \quad (\text{as } \sin \theta = 0, \text{ then } \sin 3\theta = 0)$$

$$\Rightarrow \sum_{i=1}^{10} \sin \theta_i = \frac{1}{3} \sum_{i=1}^{10} \sin 3\theta_i \leq \frac{10}{3}$$

12. (c) Given expression = $\frac{\sin 50^\circ + \sin 140^\circ + \sin 170^\circ}{2 \sin 25^\circ \sin 70^\circ \sin 85^\circ}$

$$= \frac{4}{2} \quad (\text{Using } \sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C,$$

$$\text{where } A + B + C = 180^\circ)$$

$$= 2$$

CHAPTER 4

DPP 4.1

Single Correct Answer Type

1. (c) $3^{2(1+\tan^2 x)} + 1 - 10 \cdot 3^{\tan^2 x} = 0$

$$\Rightarrow 3^2(3^{\tan^2 x})^2 + 1 - 10 \cdot 3^{\tan^2 x} = 0$$

Put $3^{\tan^2 x} = t$

$$9t^2 - 10t + 1 = 0$$

$$\Rightarrow t = \frac{1}{9} \text{ and } t = 1$$

$$\Rightarrow 3^{\tan^2 x} = 3^{-2} \text{ and } 3^{\tan^2 x} = 1$$

$$\Rightarrow \tan^2 x = -2 \text{ gives no solution and } \tan^2 x = 1 \text{ gives four solutions}$$

2. (a) $\sin^2 \theta = k$ gives four roots

$$\theta, \pi - \theta, \pi + \theta \text{ and } 2\pi - \theta$$

$$\Rightarrow \text{the sum of the roots} = 4\pi = (n-2)\pi \Rightarrow n = 6.$$

3. (c) $\tan 2x (\tan 2x + \tan 3x) = 1 - \tan 2x \tan 3x$

$$\Rightarrow \tan 2x \tan 5x = 1$$

$$\Rightarrow \cos 7x = 0$$

$$\therefore x = (2k+1)\frac{\pi}{14}, k \in \mathbb{Z}$$

$$\therefore x = \frac{\pi}{14}, \frac{3\pi}{14}, \frac{5\pi}{14}$$

4. (a) Given equation

$$2 \cos^2 x - 3 \cos x = \frac{\sin x \sin 2x}{(\cos 2x \cdot \sin x - \sin 2x \cdot \cos x) \cdot (-\sin x)}$$

$$\therefore 2 \cos^2 x - 3 \cos x = \frac{2 \sin^2 x \cos x}{(-\sin x)(-\sin x)} = 2 \cos x$$

$$\therefore 2 \cos^2 x - 5 \cos x = 0$$

$$\therefore \cos x = 0 \text{ or } \cos x = \frac{5}{2}$$

$$\text{We must have } x \neq n\pi, x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

Hence, no solution.

5. (b) $\sqrt{\sin x} - \frac{1}{\sqrt{\sin x}} = \cos x$

$$\text{Squaring, } \sin x - 2 + \frac{1}{\sin x} = \cos^2 x$$

$$\Rightarrow \sin x - 2 + \frac{1}{\sin x} = 1 - \sin^2 x$$

$$\Rightarrow \sin^2 x - 3 \sin x + 1 = -\sin^3 x$$

$$\Rightarrow \sin^3 x + \sin^2 x - 3 \sin x + 1 = 0$$

$$\Rightarrow (\sin^2 x + 2 \sin x - 1)(\sin x - 1) = 0$$

$$\Rightarrow \sin x = 1, \sin x = -1 \pm \sqrt{2}$$

$$\Rightarrow x = \pi/2, x = \pi - \sin^{-1}(-1 + \sqrt{2}) \text{ as } (\cos x \text{ must be } < 0)$$

6. (b) $(\sin^2 x + \sin x + 1)(\sin x - 2 \cos x) = 0$

$$\Rightarrow \tan x = 2$$

7. (c) $\sqrt{13 - 18 \tan x} = 6 \tan x - 3 - 2\pi < x < 2\pi$ (given)

$$\Rightarrow 13 - 18 \tan x = 36 \tan^2 x + 9 - 36 \tan x$$

$$\Rightarrow 36 \tan^2 x - 18 \tan x - 4 = 0$$

$$\Rightarrow 36 \tan^2 x - 24 \tan x + 6 \tan x - 4 = 0$$

$$\Rightarrow 6 \tan x (6 \tan x + 1) - 4(6 \tan x + 1) = 0$$

$$\tan x = \frac{2}{3} \text{ or } \tan x = -1/6 \text{ (not possible)}$$

Let $\tan \alpha = 2/3$

then $\tan x = \tan \alpha$

$$\text{or } x = \tan^{-1}(2/3), \pi + \tan^{-1}(2/3), \tan^{-1}(2/3) - \pi, \tan^{-1}(2/3) - 2\pi$$

8. (a) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 + \sin \theta & 1 \\ 1 & 1 & 1 + \cot \theta \end{vmatrix} = 0$

$$\therefore \sin \theta \cdot \cot \theta = 0$$

$$\Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

9. (c) **Case I:** $\cos x \geq 0$

$$\therefore \cos x = \cos x - 2 \sin x$$

$$\therefore \sin x = 0$$

$$\therefore x = 0, 2\pi, 4\pi, 6\pi$$

Case II: $\cos x < 0$

$$\therefore \tan x = 1$$

$$\therefore x \text{ lies in the third quadrant.}$$

$$\therefore x = \frac{5\pi}{4}, \frac{5\pi}{4} + 2\pi, \frac{5\pi}{4} + 4\pi$$

10. (c) $\log_5 \tan \theta = \log_5 4 \cdot \log_4 (3 \sin \theta)$

$$\Rightarrow \log_5 \tan \theta = \log_5 (3 \sin \theta)$$

$$\Rightarrow \tan \theta = 3 \sin \theta$$

$$\Rightarrow \sin \theta = 0 \text{ or } \cos \theta = 1/3$$

$$\sin \theta = 0 \text{ is not possible}$$

$$\therefore \cos \theta = 1/3$$

$$\text{But } \sin \theta, \tan \theta > 0$$

$$\therefore \text{for } \cos \theta = 1/3, \theta \text{ lies in the first quadrant only}$$

$$\therefore \text{four solutions only}$$

11. (c) $\log_{10}(\sin x) + \log_{10}(\tan y) + \log_{10} 2 = 0$

$$\therefore 2 \sin x \tan y = 1$$

$$\text{where } \sin x > 0, \tan y > 0$$

$$\cot y = 2\sqrt{3} \cos x$$

$$\text{From (1) and (2), } 2 \sin x = 2\sqrt{3} \cos x$$

$$\therefore \tan x = \sqrt{3}$$

$$\Rightarrow x = 2n\pi + \frac{\pi}{3}, n \in \mathbb{I} \text{ (as } \sin x > 0)$$

$$\text{From (1), } \tan y = \frac{1}{\sqrt{3}} \Rightarrow y = n\pi + \frac{\pi}{6}, n \in \mathbb{I}$$

Hence, ordered pairs are $(\pi/3, \pi/6), (\pi/3, 7\pi/6), (7\pi/3, \pi/6), (7\pi/3, 7\pi/6)$.

12. (b) $3^{\sin 2x + 2 \cos^2 x} + 3^{1 + 2(1 - \cos^2 x) - \sin 2x} = 28$

$$\Rightarrow 3^{\sin 2x + 2 \cos^2 x} + \frac{27}{3^{\sin 2x + 2 \cos^2 x}} = 28$$

$$\Rightarrow (3^t)^2 - 28(3^t) + 27 = 0 \Rightarrow 3^t = 27 \text{ or } 1$$

$$\Rightarrow \sin 2x + 2 \cos^2 x = 0 \text{ or } 3 \text{ (not possible)}$$

$$\Rightarrow \sin 2x + 2 \cos^2 x = 0$$

$$\Rightarrow \sin 2x + \cos 2x = -1$$

$$\Rightarrow \sin 4x = 0 \text{ (by squaring)}$$

$$\Rightarrow 4x = n\pi, n \in \mathbb{Z}$$

$$\Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}$$

13. (a) $a \sin^3 x + (b - a) \sin^2 x + (c - b) \sin x - c = 0$

$$\Rightarrow (\sin x - 1)(a \sin^2 x + b \sin x + c) = 0$$

$$\text{Also } a + b + c = 0$$

$\therefore$ one of the solutions of the equation $a \sin^2 x + b \sin x + c = 0$ is $\sin x = 1$

$$\text{Product of roots} = c/a$$

For three solutions, we must have $0 \leq \frac{c}{a} < 1$

14. (b) $2 \sin\left(2x + \frac{\pi}{18}\right) \cdot \cos\left(2x - \frac{2\pi}{18}\right) = \frac{-1}{2}$

$$\therefore 2 \sin(2x + 10^\circ) \cos(2x - 20^\circ) = \frac{-1}{2}$$

$$\therefore \sin(4x - 10^\circ) + \sin 30^\circ = \frac{-1}{2}$$

$$\therefore \sin(4x - 10^\circ) = -1 = \sin 270^\circ$$

Now period of $\sin(4x - 10^\circ)$ is $\pi/2$

For $x \in (0, \pi/2)$

$$\therefore 4x = 280^\circ$$

$$\therefore x = 70^\circ$$

$\therefore$ for $x \in [0, 2\pi]$, there are four solutions

15. (c) $\sin 2\theta = \cos \theta$ or $2 \sin \theta \cos \theta - \cos \theta = 0$

$$\Rightarrow \cos \theta (2 \sin \theta - 1) = 0$$

$$\Rightarrow \cos \theta = 0 \text{ or } \sin \theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \text{ all four satisfying first equation.}$$

16. (a) $\sin \theta = k_1$ has maximum roots if $k_1 = 0$, so $\theta = \pi, \pi, 2\pi$

$\cos \theta = k_2$ has maximum roots if $k_2 = \pm 1$, so there are two values of θ

$$\therefore n_1 = 3 \text{ and } n_2 = 2$$

$$\therefore 7\theta = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z} \text{ or } \theta = (2n+1)\frac{\pi}{14}, n \in \mathbb{Z}$$

2. $3 - 2 \cos \theta - 4 \sin \theta - \cos 2\theta + \sin 2\theta = 0$

$$\Rightarrow 3 - 2 \cos \theta - 4 \sin \theta - 1 + 2 \sin^2 \theta + 2 \sin \theta \cos \theta = 0$$

$$\Rightarrow 2 + 2 \sin^2 \theta - 4 \sin \theta + 2 \cos \theta (\sin \theta - 1) = 0$$

$$\Rightarrow 2(\sin^2 \theta - 2 \sin \theta + 1) + 2 \cos \theta (\sin \theta - 1) = 0$$

$$\Rightarrow 2(\sin \theta - 1)^2 + 2 \cos \theta (\sin \theta - 1) = 0$$

$$\Rightarrow (\sin \theta - 1)(\sin \theta - 1 + \cos \theta) = 0$$

$$\Rightarrow \sin \theta = 1 \text{ or } \sin \theta + \cos \theta = 1$$

$$\Rightarrow \theta = (4n+1)\frac{\pi}{2} \text{ or } \theta = 2n\pi, n \in \mathbb{Z}$$

3. $3^{\left(\frac{1}{2} + \log_3(\cos x + \sin x)\right)} - 2^{\log_2(\cos x - \sin x)} = \sqrt{2}$

$$\text{or } \sqrt{3}(\cos x + \sin x) - (\cos x - \sin x) = \sqrt{2}$$

$$\text{or } \sqrt{3} \cos x + \sqrt{3} \sin x - \cos x + \sin x = \sqrt{2}$$

$$\text{or } (\sqrt{3} + 1) \sin x + (\sqrt{3} - 1) \cos x = \frac{2\sqrt{2}}{2}$$

$$\text{or } \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) \sin x + \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) \cos x = \frac{1}{2}$$

$$\text{or } \sin\left(x + \frac{5\pi}{12}\right) = \frac{1}{2}$$

$$\text{or } x + \frac{5\pi}{12} = 2n\pi \pm \frac{\pi}{6}, n \in \mathbb{Z}$$

$$\text{or } x = 2n\pi \pm \frac{\pi}{3} - \frac{5\pi}{12}$$

$$\text{or } x = 2n\pi + \frac{3\pi}{4}, 2n\pi + \frac{\pi}{12}, n \in \mathbb{Z}$$

4. $\cos 3x \cdot \cos^3 x + \sin 3x \cdot \sin^3 x = 0$

$$\therefore (4 \cos^3 x - 3 \cos x) \cos^3 x + (3 \sin x - 4 \sin^3 x) \sin^3 x = 0$$

$$\therefore 4(\cos^6 x - \sin^6 x) - 3(\cos^4 x - \sin^4 x) = 0$$

$$\therefore 4(\cos^6 x - \sin^6 x) - 3(\cos^2 x - \sin^2 x) = 0$$

$$\therefore (\cos^2 x - \sin^2 x)(1 - 4 \cos^2 x \sin^2 x) = 0$$

$$\therefore \cos 2x = 0 \text{ or } \sin^2 2x = 1 = \sin \pi/2$$

$$\therefore 2x = (2n+1)\pi/2 \text{ or } 2x = n\pi \pm \pi/2$$

$$\therefore x = (2n+1)\pi/4 \text{ or } x = \frac{n\pi}{2} \pm \frac{\pi}{4}, n \in \mathbb{Z}$$

$$\therefore x = \frac{n\pi}{2} \pm \frac{\pi}{4}, n \in \mathbb{Z}$$

5. $4 \cos^2 x \sin x - 2 \sin^2 x = 3 \sin x$

$$\Rightarrow 4(1 - \sin^2 x) \sin x - 2 \sin^2 x - 3 \sin x = 0$$

$$\Rightarrow \sin x(4 - 4 \sin^2 x - 2 \sin x - 3) = 0$$

$$\Rightarrow \sin x(4 \sin^2 x + 2 \sin x - 1) = 0$$

$$\Rightarrow \sin x = 0 \text{ or } 4 \sin^2 x + 2 \sin x - 1 = 0$$

$$\Rightarrow x = n\pi, n \in \mathbb{Z} \text{ or } x = \frac{\sqrt{5}-1}{4}, \frac{-(\sqrt{5}+1)}{4}$$

$$\Rightarrow x = n\pi \text{ or } x = n\pi + (-1)^n \frac{\pi}{10} \text{ or } x = n\pi + (-1)^n \left(-\frac{3\pi}{10}\right), n \in \mathbb{Z}$$

6. $1 + 2 \operatorname{cosec} x = -\frac{\sec^2 x}{2}$

DPP 4.2

Subjective Type

1. $\frac{1}{\cos \theta} + \frac{1}{\cos 3\theta} = \frac{1}{\cos 5\theta}$

$$\Rightarrow (\cos 3\theta + \cos \theta) \cos 5\theta = \cos 3\theta \cos \theta$$

$$\Rightarrow 2 \cos 5\theta \cos 3\theta + 2 \cos 5\theta \cos \theta = 2 \cos 3\theta \cos \theta$$

$$\Rightarrow \cos 8\theta + \cos 2\theta + \cos 6\theta + \cos 4\theta = \cos 4\theta + \cos 2\theta$$

$$\Rightarrow 2 \cos 7\theta \cos \theta = 0$$

$$\Rightarrow \cos 7\theta = 0 \text{ (as } \cos \theta = 0 \text{ is not possible)}$$

$$\therefore \cos 7\theta = 0$$

S.22 Trigonometry

$$\Rightarrow 1 + 2 \left(\frac{1 + \tan^2 \frac{x}{2}}{2 \tan \frac{x}{2}} \right) = - \frac{(1 + \tan^2 \frac{x}{2})}{2}$$

$$\Rightarrow 2 \tan \frac{x}{2} + 2 + 2 \tan^2 \frac{x}{2} + \tan \frac{x}{2} + \tan^3 \frac{x}{2} = 0$$

$$\Rightarrow \tan^3 \frac{x}{2} + 2 \tan^2 \frac{x}{2} + 3 \tan \frac{x}{2} + 2 = 0$$

$$\Rightarrow \left(\tan \frac{x}{2} + 1 \right) \left(\tan^2 \frac{x}{2} + \tan \frac{x}{2} + 2 \right) = 0$$

$$\Rightarrow \tan \left(\frac{x}{2} \right) + 1 = 0$$

$$\Rightarrow \tan \frac{x}{2} = \tan \left(-\frac{\pi}{4} \right)$$

$$\Rightarrow x = 2n\pi - \frac{\pi}{2}, n \in \mathbb{Z}$$

7. $2 \sin \left(3x + \frac{\pi}{4} \right) = \sqrt{1 + 8 \sin 2x \cdot \cos^2 2x}$

$$\Rightarrow 2 \left(\frac{\sin 3x + \cos 3x}{\sqrt{2}} \right) = \sqrt{1 + 8 \sin 2x \cos 2x \cos 2x}$$

$$\Rightarrow \sqrt{2} (\sin 3x + \cos 3x) = \sqrt{1 + 4 \sin 4x \cos 2x}$$

$$\Rightarrow 2(\sin 3x + \cos 3x)^2 = 1 + 2(\sin 6x + \sin 2x)$$

$$\Rightarrow 2(1 + \sin 6x) = 1 + 2 \sin 6x + 2 \sin 2x$$

$$\Rightarrow 2 \sin 2x = 1$$

$$\Rightarrow \sin 2x = 1/2 = \sin \pi/6$$

$$\text{or } 2x = n\pi + (-1)^n \pi/6, n \in \mathbb{Z}$$

$$\text{or } x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12}, n \in \mathbb{Z}$$

$$\therefore x = (6n + (-1)^n) \frac{\pi}{12}$$

$$\therefore x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

But for $x = \frac{5\pi}{12}$ and $\frac{13\pi}{12}$;

$$\sin \left(3x + \frac{\pi}{4} \right) < 0$$

$$\therefore x = \frac{\pi}{12}, \frac{17\pi}{12}$$

8. $5^{(\csc^2 x - 3 \sec^2 y)} = 1 = 5^0$

$$2^{(2 \csc x + \sqrt{3} |\sec y|)} = 64 = 2^6$$

$$\Rightarrow \csc^2 x - 3 \sec^2 y = 0$$

$$\text{And } 2 \csc x + \sqrt{3} |\sec y| = 6$$

$$\text{Putting the value of } \sqrt{3} |\sec y| \text{ from equation (2) in (1)}$$

$$\csc^2 x - (6 - 2 \csc x)^2 = 0$$

$$\Rightarrow \csc^2 x - 36 + 24 \csc x - 4 \csc^2 x = 0$$

$$\Rightarrow 3 \csc^2 x - 24 \csc x + 36 = 0$$

$$\Rightarrow \csc^2 x - 8 \csc x + 12 = 0$$

$$\Rightarrow \csc x = 2, 6$$

$$\text{Putting the value of } \csc x \text{ in equation (2), we get}$$

$$|\sec y| = \frac{2}{\sqrt{3}}$$

$$\Rightarrow y = m\pi \pm \frac{\pi}{6}, m \in \mathbb{Z}$$

$$\Rightarrow x = n\pi + (-1)^n \pi/6, n \in \mathbb{Z}$$

9. $2 + \tan x \cdot \cot \frac{x}{2} + \cot x \cdot \tan \frac{x}{2} = 0$

$$\text{Let } \tan x \cdot \cot \frac{x}{2} = y$$

Then equation (1) becomes

$$2 + \left(y + \frac{1}{y} \right) = 0$$

$$\Rightarrow y + \frac{1}{y} = -2$$

On solving, we get

$$y = -1$$

$$\text{or } \tan x \cdot \cot \frac{x}{2} = -1$$

$$\text{or } \frac{\sin x \cos \frac{x}{2}}{\cos x \sin \frac{x}{2}} = -1$$

$$\text{or } \frac{2 \sin \frac{x}{2} \cos \frac{x}{2} \cdot \cos \frac{x}{2}}{\cos x \sin \frac{x}{2}} = -1$$

$$\text{or } \frac{2 \cos^2 \frac{x}{2}}{\cos x} = -1$$

$$\text{or } \frac{1 + \cos x}{\cos x} = -1$$

$$\text{or } \cos x = -\frac{1}{2}$$

$$\text{or } x = 2n\pi \pm \frac{2\pi}{3}, n \in \mathbb{Z}$$

Single Correct Answer Type

10. (c) $\frac{\sin 3\theta}{2 \cos 2\theta + 1} = \frac{1}{2}$

$$\Rightarrow \frac{3 \sin \theta - 4 \sin^3 \theta}{3 - 4 \sin^2 \theta} = \frac{1}{2}$$

$$\Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = n\pi + (-1)^n \pi/6, n \in \mathbb{Z}$$

(1) 11. (a) The equation

(2)

$$\frac{1 - \sin x + \dots + (-1)^n \sin^n x + \dots}{1 + \sin x + \dots + \sin^n x + \dots} = \frac{1 - \cos 2x}{1 + \cos 2x}$$

$$\Rightarrow \frac{1}{1 + \sin x} \times \frac{1 - \sin x}{1} = \frac{2 \sin^2 x}{2 \cos^2 x}$$

$$\Rightarrow \frac{1 - \sin x}{1 + \sin x} = \frac{\sin^2 x}{1 - \sin^2 x}$$

$$\Rightarrow (1 - \sin x)^2 = \sin^2 x$$

$$\Rightarrow \sin x = \frac{1}{2}$$

$$\Rightarrow \sin x = \sin(\pi/6)$$

$$\Rightarrow x = n\pi + (-1)^n(\pi/6), n \in \mathbb{Z}$$

$$12. (b) 2 \cot \frac{\theta}{2} = (1 + \cot \theta)^2$$

$$\Rightarrow \frac{2 \cdot 2 \cos \theta/2 \cdot \cos \theta/2}{2 \sin \theta/2 \cdot \cos \theta/2} = (1 + \cot \theta)^2$$

$$\Rightarrow \frac{2(1 + \cos \theta)}{\sin \theta} = \operatorname{cosec}^2 \theta + 2 \cot \theta$$

$$\Rightarrow 2 + 2 \cos \theta = \operatorname{cosec} \theta + 2 \cos \theta$$

$$\Rightarrow \sin \theta = 1/2$$

$$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{6}, n \in \mathbb{Z}$$

$$13. (d) \cos 2\theta = (\sqrt{2} + 1) \left(\cos \theta - \frac{1}{\sqrt{2}} \right)$$

$$\Rightarrow (2 \cos^2 \theta - 1) = \frac{\sqrt{2} + 1}{\sqrt{2}} (\sqrt{2} \cos \theta - 1)$$

$$\Rightarrow (\sqrt{2} \cos \theta - 1)(\sqrt{2} \cos \theta + 1) = \frac{\sqrt{2} + 1}{\sqrt{2}} (\sqrt{2} \cos \theta - 1)$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{2}} \text{ or } \cos \theta = \frac{1}{2}$$

$$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{4} \text{ or } \theta = 2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$

$$14. (a) 16(\sin^5 x + \cos^5 x) - 11(\sin x + \cos x) = 0$$

$$\Rightarrow (\sin x + \cos x) \{ 16(\sin^4 x - \sin^3 x \cos x + \sin^2 x \cos^2 x - \sin x \cos^3 x + \cos^4 x) - 11 \} = 0$$

$$\Rightarrow (\sin x + \cos x) \{ 16(1 - \sin^2 x \cos^2 x - \sin x \cos x) - 11 \} = 0$$

$$\Rightarrow (\sin x + \cos x)(4 \sin x \cos x - 1)(4 \sin x \cos x + 5) = 0$$

As $4 \sin x \cos x + 5 \neq 0$, we have

$$\sin x + \cos x = 0, 4 \sin x \cos x - 1 = 0$$

$$\Rightarrow \tan x = -1, \sin 2x = \frac{1}{2}$$

$$\Rightarrow \pi/12, 5\pi/12, 9\pi/12, 13\pi/12, 17\pi/12, 21\pi/12.$$

There are 6 solutions on $[0, 2\pi]$.

$$15. (d) \sin \pi x + \cos \pi x = 0$$

$$\text{or } \sin\left(\frac{\pi}{4} + \pi x\right) = 0$$

$$\text{or } \frac{\pi}{4} + \pi x = n\pi, n \in \mathbb{Z}$$

$$\text{or } x = n - \frac{1}{4}, \text{ where } n \in \mathbb{Z}.$$

$$\text{Sum} = \sum_{n=1}^{100} \left(n - \frac{1}{4} \right)$$

$$= \frac{100 \times 101}{2} - \frac{100}{4}$$

$$= 5050 - 25$$

$$= 5025$$

$$16. (b) \text{ We have } [4 \cos^2 x - 2 \sin x - 3] \sin x = 0.$$

$$\Rightarrow \sin x = 0 \text{ or } 4(1 - \sin^2 x) - 2 \sin x - 3 = 0$$

$$\Rightarrow \sin x = 0 \text{ or } 4 \sin^2 x + 2 \sin x - 1 = 0$$

$$\Rightarrow \sin x = 0 \text{ or } \sin x = \frac{\sqrt{5}-1}{4}, -\frac{1+\sqrt{5}}{4}$$

$$\Rightarrow x = n\pi \text{ or } x = n\pi + (-1)^n \frac{\pi}{10}$$

$$\text{or } x = n\pi + (-1)^{n+1} \left(\frac{3\pi}{10} \right), n \in \mathbb{Z}$$

$$17. (c) \cos^2 \left(\frac{\pi}{3} \cos x - \frac{8\pi}{3} \right) = 1$$

$$\Rightarrow \frac{\pi}{3} \cos x - \frac{8\pi}{3} = k\pi, (k \in \mathbb{Z})$$

$$\Rightarrow \cos x = 3k + 8, k \in \mathbb{Z}$$

$$\Rightarrow \cos x = -1, \text{ when } k = -3$$

$$\Rightarrow \text{there are 5 solutions for } x \in [0, 10\pi]$$

Multiple Correct Answers Type

18. (a, d)

$$\cos \frac{\alpha}{2} + \sin \frac{\alpha}{2} = \sqrt{2} (\cos 36^\circ - \sin 18^\circ)$$

$$\Rightarrow \cos \left(\frac{\alpha}{2} - \frac{\pi}{4} \right) = \frac{\sqrt{5}+1}{4} - \frac{\sqrt{5}-1}{4}$$

$$\Rightarrow \cos \left(\frac{\alpha}{2} - \frac{\pi}{4} \right) = \frac{1}{2}$$

$$\Rightarrow \frac{\alpha}{2} - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$

$$\therefore \alpha = 4n\pi + \frac{\pi}{2} \pm \frac{2\pi}{3}$$

$$\therefore \alpha = 4n\pi + \frac{7\pi}{6}, 4n\pi - \frac{\pi}{6}$$

$$\text{For } n = 0, \alpha = \frac{7\pi}{6}, -\frac{\pi}{6}.$$

19. (c, d)

$$(a) \text{ is false since } \cos(n-5) \frac{\pi}{5} \neq 0 \text{ for all integers } n.$$

$$(b) \text{ is false since } \sin \frac{6}{5} \cdot 6(n-1)\pi \neq 0 \text{ for all integers } n.$$

$$(c) \text{ is correct since } \sin \frac{6}{5} x = \sin(6n-3)\pi = 0$$

$$\text{Also } \cos \frac{x}{5} = \cos \left(n - \frac{1}{2} \right) \pi = \cos(2n-1) \frac{\pi}{2} = 0$$

$$(d) \text{ is also correct since } \sin \frac{6}{5} x = \sin(6n+3)\pi = 0$$

$$\text{Also } \cos \frac{x}{5} = \cos \left(n + \frac{1}{2} \right) \pi = \cos(2n+1) \frac{\pi}{2} = 0$$

20. (a, b)

$$\text{Let } x = \cos \theta$$

$$\Rightarrow 4 \cos^3 \theta - 3 \cos \theta = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos 3\theta = \cos \frac{5\pi}{6}$$

$$\Rightarrow 3\theta = 2n\pi \pm \frac{5\pi}{6}, n \in \mathbb{Z}$$

$$\Rightarrow \theta = \frac{2n\pi}{3} \pm \frac{5\pi}{18}, n \in \mathbb{Z}$$

$$\text{Put } n = 0, \theta = \frac{5\pi}{18}$$

$$n = 1, \theta = \frac{2\pi}{3} + \frac{5\pi}{18} = \frac{17\pi}{18}$$

$$\theta = \frac{2\pi}{3} - \frac{5\pi}{18} = \frac{7\pi}{18}$$

21. (a, b)

We have $5 \sin x \cos y = 1$ and $4 \tan x = \tan y$

$$\Rightarrow 5 \sin x \cos y = 1 \text{ and } 4 \sin x \cos y = \sin y \cos x$$

$$\Rightarrow \sin x \cos y = \frac{1}{5} \text{ and } \cos x \sin y = \frac{4}{5}$$

$$\Rightarrow \sin x \cos y + \cos x \sin y = 1$$

$$\text{and } \sin x \cos y - \cos x \sin y = -\frac{3}{5}$$

$$\Rightarrow \sin(x+y) = \sin \frac{\pi}{2} \text{ and } \sin(x-y) = \sin \left\{ \sin^{-1} \left(-\frac{3}{5} \right) \right\}$$

$$\Rightarrow x+y = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$$

$$\text{and } x-y = m\pi + (-1)^m \sin^{-1} \left(-\frac{3}{5} \right), m \in \mathbb{Z}$$

$$\Rightarrow x = (m+n)\frac{\pi}{2} + \frac{\pi}{4} + (-1)^m \frac{1}{2} \sin^{-1} \left(-\frac{3}{5} \right); m, n \in \mathbb{Z}$$

$$\text{and } y = (n-m)\frac{\pi}{2} + \frac{\pi}{4} + (-1)^{m+1} \frac{1}{2} \sin^{-1} \left(-\frac{3}{5} \right); m, n \in \mathbb{Z}$$

DPP 4.3

Single Correct Answer Type

1. (c) $\sin \gamma \cdot \cos \alpha = 1 \quad \alpha, \gamma \in [\pi, 2\pi]$

$$\therefore \sin \gamma = \cos \alpha = 1$$

$$\Rightarrow \gamma = \pi/2, \alpha = 2\pi \text{ (rejected) (as } \alpha < \beta < \gamma)$$

$$\text{Other possibility is } \sin \gamma = \cos \alpha = -1 \Rightarrow \gamma = 3\pi/2, \alpha = \pi$$

$$f(x)_{\min} = f(\beta) = \beta - \alpha + 0 + \gamma - \beta$$

$$= \gamma - \alpha$$

$$= \frac{3\pi}{2} - \pi = \frac{\pi}{2}$$

$$f(x) \geq \pi/2 \Rightarrow \text{least integral value of } f(x) \text{ is } 2.$$

2. (a) $\frac{1}{5^2} + \frac{1}{5^{2+\log_5(\sin x)}} = \frac{1}{5^{2+\log_5 \cos x}}$

$$\Rightarrow \frac{1}{5^2} + \frac{1}{5^2 \cdot 5^{\log_5(\sin x)}} = \frac{1}{5^2 \cdot 5^{\log_5 \cos x}}$$

$$\Rightarrow \frac{1}{5^2} + \frac{1}{5^{2+\log_5 \sin x}} = \frac{1}{5^{2+\log_5 \cos x}}$$

$$\Rightarrow 1 + \sin x = \sqrt{3} \cos x$$

$$\Rightarrow \frac{\sqrt{3}}{2} \cos x - \frac{\sin x}{2} = \frac{1}{2}$$

$$\Rightarrow \cos \left(x + \frac{\pi}{6} \right) = \cos \frac{\pi}{3}$$

$$\Rightarrow x + \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$

$$\Rightarrow x = 2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{6}, n \in \mathbb{Z}$$

But we must have $\sin x, \cos x > 0$

$$\therefore x = 2n\pi + \pi/6, n \in \mathbb{Z}$$

3. (b) Subtracting we get, $\sin y = 1 + 2009 \cos y$

$$\therefore \text{Equation is possible if } \cos y = 0$$

$$\therefore y = \pi/2 \text{ and } x = 2008$$

$$\therefore [x+y] = 2009$$

4. (a) Given equation can hold only if $\sin \frac{5x}{2} = 1$ and $\sin \frac{x}{2} = -1$

$$\text{i.e. } \frac{5x}{2} = 2n\pi + \frac{\pi}{2} \text{ and } \frac{x}{2} = 2p\pi - \frac{\pi}{2} \quad (n \text{ and } p \in \mathbb{I})$$

For some possible p and n if there exists a solution, we must have

$$10p\pi - \frac{5\pi}{2} = 2n\pi + \frac{\pi}{2}$$

$$\text{or } 10p - 2n = 3$$

L.H.S. is even, R.H.S. is odd

Hence, not possible for any p and n .

5. (a) $\therefore \text{A.M.} \geq \text{G.M.} \therefore \frac{2^{\sin x} + 2^{\cos x}}{2} \geq \sqrt{2^{\sin x} \cdot 2^{\cos x}}$

$$\therefore 2^{\sin x} + 2^{\cos x} \geq 2 \cdot \sqrt{2^{\sin x + \cos x}}$$

But minimum value of $\cos x + \sin x$ is $-\sqrt{2}$

$$\therefore 2^{\sin x} + 2^{\cos x} \geq 2 \cdot \sqrt{2^{-\sqrt{2}}} = 2^{1-\frac{1}{\sqrt{2}}}$$

But the given equation is $2^{\sin x} + 2^{\cos x} = 2^{1-\frac{1}{\sqrt{2}}}$, which can hold

$$\text{only if } 2^{\sin x} = 2^{\cos x} = 2^{-\frac{1}{\sqrt{2}}}$$

$$\Rightarrow x = 2n\pi + \frac{5\pi}{4}, n \in \mathbb{Z}$$

6. (a) For real solution $\sqrt{k^2 + k - 2} \geq |\tan \alpha + \cot \alpha| \geq 2$

$$\Rightarrow k^2 + k - 6 \geq 0$$

$$\Rightarrow k \in (-\infty, -3] \cup [2, \infty)$$

7. (c) $x^2 + 12 + 3 \sin(a+bx) + 6x = 0$

$$\Rightarrow (x+3)^2 + 3 + 3 \sin(a+bx) = 0$$

$$\Rightarrow (x+3)^2 + 3 = -3 \sin(a+bx)$$

$$\text{L.H.S.} \geq 3 \text{ but R.H.S.} \leq 3$$

$$\text{L.H.S.} = \text{R.H.S.} = 3$$

$$\therefore x = -3 \text{ and } \sin(a+bx) = -1$$

$$\Rightarrow \sin(a-3b) = -1$$

$$\text{or } a-3b = (4n-1)\frac{\pi}{2}, n \in \mathbb{Z}$$

8. (a) $\sin x \cdot \sin 2x \cdot \sin 3x = 1$

Case I: $\sin x = 1$ and $\sin 2x = -1$ and $\sin 3x = -1$ This does not hold simultaneously; because when $\sin x = 1$, the value of $\cos x = -1/2$ **Case II:** $\sin x = -1$ and $\sin 2x = 1$ and $\sin 3x = -1$ This does not hold simultaneously; because when $\sin x = -1$, the value of $\sin 3x = 3 \sin x - 4 \sin^3 x = 1$ **Case III:** $\sin x = -1$ and $\sin 2x = -1$ and $\sin 3x = 1$ This is not possible; because $\sin x$ and $\sin 2x$ cannot be simultaneously -1 **Case IV:** $\sin x, \sin 2x, \sin 3x = 1$

Clearly, this does not hold simultaneously.

9. (c) $\cos x + \cos\left(\frac{2\pi}{3} - x\right) = \frac{3}{2}$

$$\frac{\cos x}{2} + \frac{\sqrt{3}}{2} \sin x = \frac{3}{2}$$

$$\Rightarrow \sin\left(\frac{\pi}{6} + x\right) = \frac{3}{2} \text{ (not possible)}$$

10. (b) $\sin \pi x = x^2 - x + \frac{5}{4}$

$$\Rightarrow \sin \pi x = (x - 1/2)^2 + 1$$

$$\Rightarrow \sin \pi x \leq 1 \text{ and } x^2 - x + \frac{5}{4} \geq 1$$

$$\therefore \sin \pi x = 1 \text{ and } x = \frac{1}{2}$$

11. (c) Given $2^{\sin^2 u + 3 \cos^2 v} \cdot 3^{\sin^2 w + \cos^2 x} \cdot 5^{\cos^2 y} \geq 2^4 \cdot 3^2 \cdot 5^1$

The equality holds when $\sin^2 u = \sin^2 w = \cos^2 v = \cos^2 x = \cos^2 y = 1$

$$\therefore u, w \in \left\{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}\right\} \text{ and } x, y, v \in \{\pi, 2\pi, 3\pi\}$$

$$\text{Number of ordered 5-tuples} = 4^2 \times 3^3 = 432$$

12. (d) We must have $0 \leq p \leq 2$

$$\text{Also } \sqrt{2} + \sqrt{2-p} \leq \sqrt{p+4}$$

$$\Rightarrow 2\sqrt{4-2p} \leq 2p$$

$$\Rightarrow 4 - 2p \leq p^2$$

$$\Rightarrow p^2 + 2p - 4 \leq 0$$

$$\Rightarrow p \in [\sqrt{5} - 1, 2]$$

Multiple Correct Answers Type

13. (a, b)

$$2 \sin 11x + \cos 3x + \sqrt{3} \sin 3x = 0$$

$$\Rightarrow \frac{1}{2} \cos 3x + \frac{\sqrt{3}}{2} \sin 3x + \sin 11x = 0$$

$$\Rightarrow \sin(30^\circ + 3x) + \sin 11x = 0$$

$$\Rightarrow 2 \sin\left(\frac{\pi}{12} + 7x\right) \cos\left(\frac{\pi}{12} - 4x\right) = 0$$

$$\Rightarrow \sin\left(\frac{\pi}{12} + 7x\right) = 0 \text{ or } \cos\left(\frac{\pi}{12} - 4x\right) = 0$$

$$\Rightarrow \frac{\pi}{12} + 7x = n\pi \text{ or } 4x - \frac{\pi}{12} = n\pi + \frac{\pi}{2}, n \in \mathbb{Z}$$

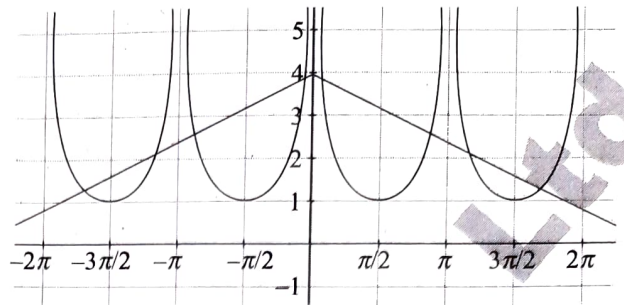
$$\Rightarrow x = \frac{n\pi}{7} - \frac{\pi}{84} \text{ or } x = \frac{n\pi}{4} + \frac{7\pi}{48}, n \in \mathbb{Z}$$

DPP 4.4

Single Correct Answer Type

1. (a) $\operatorname{cosec} x = \frac{5\pi}{4} - \left|\frac{x}{2}\right|$

Draw the graphs of $y = \operatorname{cosec} x$ and $y = \frac{5\pi}{4} - \left|\frac{x}{2}\right|$



From the graph, there are 8 solutions.

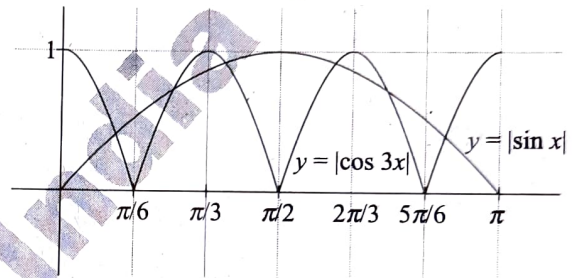
2. (c) We have $|\sin x| = |\cos 3x|$

Period of $|\sin x|$ is π and period of $|\cos 3x|$ is $\pi/3$

L.C.M. of π and $\pi/3$ is π

So let us check the roots of equation for $x \in [0, \pi]$

Draw the graphs of $y = |\sin x|$ and $y = |\cos 3x|$ for $x \in [0, \pi]$

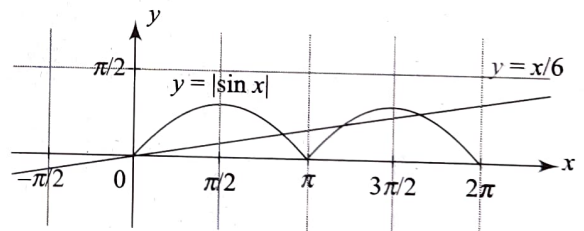


From the graph, there are 6 solutions.

So for $x \in [-2\pi, 2\pi]$, there are 24 solutions.

3. (d) We have $|\sin x| = \frac{x}{6}$

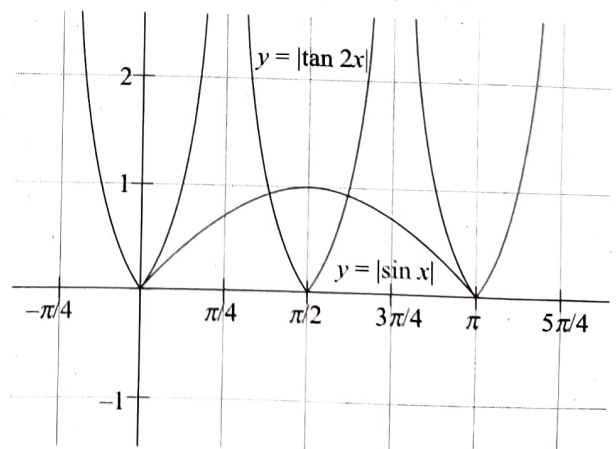
Draw the graph of $y = |\sin x|$ and $y = \frac{x}{6}$



From the graph, there are four solutions between 0 and π .

4. (b) $|\tan 2x| = \sin x$ in $[0, \pi]$

Draw the graph of $y = |\tan 2x|$ and $y = \sin x$



From the graph, there are 4 solutions.

5. (d) We have $x = \left(\frac{5\pi}{2}\right)^{\cos x}$

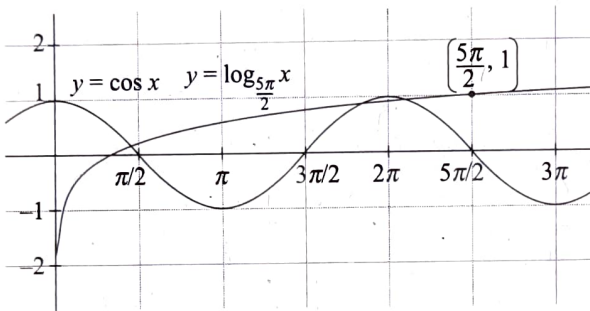
$$\therefore \log_{\frac{5\pi}{2}} x = \cos x$$

If $\log_{\frac{5\pi}{2}} x = 1$

$$\therefore x = \frac{5\pi}{2}$$

Thus, graph of $y = \log_{\frac{5\pi}{2}} x$ passes through the point $\left(\frac{5\pi}{2}, 1\right)$

$\therefore$ Draw the graph of $y = \log_{\frac{5\pi}{2}} x$ and $y = \cos x$ as shown in the following figure.



From the graph, there are 3 solutions.

6. (c) Let $\frac{\sin 3\theta}{\cos 2\theta} > 0$

Case I. $\sin 3\theta < 0$ and $\cos 2\theta > 0$

$$\Rightarrow 3\theta \in (\pi, 2\pi) \text{ and } 2\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\Rightarrow \theta \in \left(\frac{\pi}{3}, \frac{2\pi}{3}\right) \text{ and } \theta \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$$

Case II. $\sin 3\theta > 0$ and $\cos 2\theta < 0$

$$\therefore 3\theta \in (0, \pi) \text{ and } 2\theta \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$$

$$\Rightarrow \theta \in \left(0, \frac{\pi}{3}\right) \text{ and } \theta \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$$

From case I and II, we have

$$\theta \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$$

$$\text{Since } \left(\frac{13\pi}{48}, \frac{7\pi}{24}\right) \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$$

$$\therefore \theta \in \left(\frac{13\pi}{48}, \frac{7\pi}{24}\right)$$

7. (d) $2 \sin^2\left(x - \frac{\pi}{3}\right) - 5 \sin\left(x - \frac{\pi}{3}\right) + 2 < 0$

$$\Rightarrow \left(2 \sin\left(x - \frac{\pi}{3}\right) - 1\right)\left(\sin\left(x - \frac{\pi}{3}\right) - 2\right) < 0$$

$$\Rightarrow \frac{1}{2} < \sin\left(x - \frac{\pi}{3}\right) < 2$$

$$\Rightarrow \frac{1}{2} < \sin\left(x - \frac{\pi}{3}\right) \leq 1$$

$$\therefore x - \frac{\pi}{3} \in \left(2n\pi + \frac{\pi}{6}, 2n\pi + \frac{5\pi}{6}\right)$$

$$x \in \left(2n\pi + \frac{\pi}{2}, 2n\pi + \frac{7\pi}{6}\right), n \in \mathbb{I}$$

8. (c) $x^2 + 2x + 2 + e^\alpha - 2 \sin \beta = 0$ has real solution

$$\therefore D = (2)^2 - 4\{2 + e^\alpha - 2 \sin \beta\} \geq 0$$

$$\therefore 1 - 2 - e^\alpha + 2 \sin \beta \geq 0$$

$$\therefore 2 \sin \beta \geq 1 + e^\alpha$$

$$\therefore \sin \beta \geq \frac{1}{2} + \frac{1}{2}e^\alpha$$

$$\Rightarrow \alpha \leq 0$$

$$\Rightarrow \frac{1}{2} \leq \sin \beta < 1$$

$$\Rightarrow \beta \in (2n\pi + \pi/6, 2n\pi + 5\pi/6), n \in \mathbb{Z}$$

Multiple Correct Answers Type

9. (b, c)

$$f(x) = 2\left(\frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x\right) = 2 \sin\left(x - \frac{\pi}{6}\right)$$

$$\Rightarrow f(x) \text{ will increase monotonically if } -\frac{\pi}{2} \leq x - \frac{\pi}{6} \leq \frac{\pi}{2} \text{ and}$$

$$\frac{3\pi}{2} \leq x - \frac{\pi}{6} \leq \frac{5\pi}{2}$$

$$\Rightarrow -\frac{\pi}{3} \leq x \leq \frac{2\pi}{3} \text{ and } \frac{5\pi}{3} \leq x \leq \frac{8\pi}{3}$$

10. (a, b)

$$\cos 3\theta + \sin 3\theta + (2 \sin 2\theta - 3)(\sin \theta - \cos \theta) > 0$$

$$\Rightarrow 4(\cos \theta - \sin \theta)(\cos^2 \theta + \sin \theta \cos \theta + \sin^2 \theta - \sin \theta \cos \theta) > 0$$

$$\Rightarrow \sin\left(\theta - \frac{\pi}{4}\right) < 0$$

$$\Rightarrow \sin\left(\theta - \frac{\pi}{4}\right) \text{ is negative when}$$

$$\Rightarrow (2n-1)\pi < \theta - \pi/4 < 2n\pi, n \in \mathbb{Z}$$

$$\Rightarrow 2n\pi - \frac{3\pi}{4} < \theta < 2n\pi + \pi/4, n \in \mathbb{Z}$$

CHAPTER 5

DPP 5.1

Single Correct Answer Type

1. (a) $(\sin^{-1}x)^2 - \frac{\pi}{2}\sin^{-1}x + \frac{\pi^2}{18} < 0$

$$\Rightarrow (\sin^{-1}x - \pi/3)(\sin^{-1}x - \pi/6) < 0$$

$$\Rightarrow \frac{\pi}{6} < \sin^{-1}x < \frac{\pi}{3}$$

$$\Rightarrow \frac{1}{2} < x < \frac{\sqrt{3}}{2}$$

Similarly

$$\frac{\pi}{6} < \tan^{-1}x < \frac{\pi}{3}$$

$$\Rightarrow \frac{1}{\sqrt{3}} < x < \sqrt{3}$$

From (1) and (2), we get $\frac{1}{\sqrt{3}} < x < \frac{\sqrt{3}}{2}$

2. (b) Clearly $f(x)$ is defined only for $x = \pm 1$

$$\text{and } f(1) = f(-1) = \frac{\pi}{2}$$

3. (b) We have $\sin^{-1}\sqrt{x} < \frac{\pi}{4}$

$$\therefore 0 \leq x \leq 1$$

$$\text{Also, } \sqrt{x} < \frac{1}{\sqrt{2}} \Rightarrow x < \frac{1}{2}$$

From (1) and (2), we get

$$\Rightarrow x \in \left[0, \frac{1}{2}\right)$$

4. (a) $(\sin^{-1}x) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\therefore (\sin^{-1}x)^2 \leq \frac{\pi^2}{4}$$

$$(\sec^{-1}y)^2, (\tan^{-1}z)^2 \geq 0$$

$$\therefore \text{R.H.S.} \geq \frac{\pi^2}{4}$$

$$\therefore (\sin^{-1}x)^2 = \frac{\pi^2}{4}$$

$$\therefore (\sec^{-1}y)^2 + (\tan^{-1}z)^2 = 0$$

$$\text{or } \sec^{-1}y = \tan^{-1}z = 0$$

$$\therefore \sin^{-1}x = \pm \frac{\pi}{2}, y = 1, z = 0$$

$$\therefore x = \pm 1, y = 1, z = 0$$

5. (c) We have $f(x) = \sin^{-1}(x - \sqrt{x})$

$$= \sin^{-1}\left(\left(\sqrt{x} - \frac{1}{2}\right)^2 - \frac{1}{4}\right)$$

$$\therefore \text{Range of } f(x) \text{ is } \left[\sin^{-1}\left(-\frac{1}{4}\right), \sin^{-1}\left(\frac{1}{4}\right)\right]$$

$$= \left[-\sin^{-1}\frac{1}{4}, \frac{\pi}{2}\right]$$

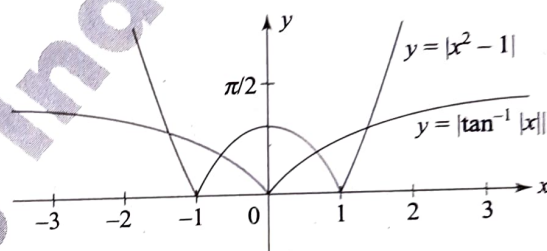
6. (c) $\sqrt{(x^2+1)^2 - 4x^2} = \sqrt{(x^2-1)^2} = |x^2-1|$

$$\Rightarrow |\tan^{-1}|x|| = |x^2-1|$$

Draw the graphs of $y = |\tan^{-1}|x||$ and $y = |x^2-1|$

(1)

(2)



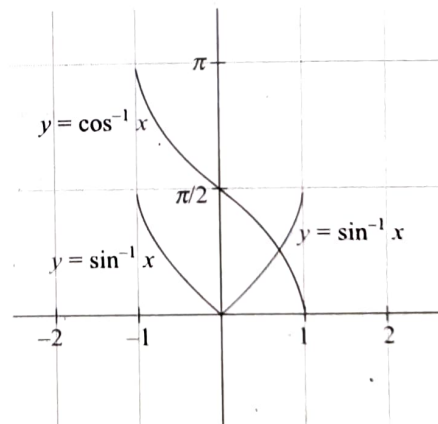
From the graph, it is clear that equation has four roots.

7. (b) $\sin^{-1}|x| = |\cos^{-1}x|$

$$\text{or } \sin^{-1}|x| = \cos^{-1}x$$

$$\text{or } \begin{cases} \sin^{-1}x = \cos^{-1}x, & x \geq 0 \\ -\sin^{-1}x = \cos^{-1}x, & x < 0 \end{cases}$$

Draw the graph of $y = \sin^{-1}|x|$ and $y = \cos^{-1}x$



From the graph, there is only one solution.

8. (c) For $x \in (0, 1)$

$$\sin^{-1}x > x > \tan^{-1}x > \cot^{-1}x - \frac{\pi}{2}$$

$$\text{as } \cot^{-1}x - \frac{\pi}{2} = -(\tan^{-1}x)$$

S.28 Trigonometry

9. (b) Let $\tan^{-1} x = \alpha \Rightarrow x = \tan \alpha$
 $\tan^{-1} y = \beta \Rightarrow y = \tan \beta$
 $\tan^{-1} z = \gamma \Rightarrow z = \tan \gamma$

$\therefore \alpha + \beta + \gamma = \frac{\pi}{4}$

and $x + y + z = 1$ (Given)
 $\tan(\alpha + \beta + \gamma) = 1$

$\Rightarrow \frac{x + y + z - xyz}{1 - xy - yz - zx} = 1$

$\Rightarrow x + y + z - xyz = 1 - xy - yz - zx$

$\Rightarrow 1 - xyz = 1 - xy - yz - zx$

$\Rightarrow xyz = xy + yz + zx$

$\therefore xyz - xy - yz - zx + x + y + z - 1 = 0$
 $(\because x + y + z - 1 = 0)$

$\Rightarrow (x-1)(y-1)(z-1) = 0$

If $x = 1$, then $y + z = 0$

$\therefore x^3 + y^3 + z^3 - 3 = 1^3 + y^3 - y^3 - 3 = -2$

10. (a) $\tan^{-1}(x - 18x + a) > 0 \forall x \in \mathbb{R}$

$\Rightarrow x^2 - 18x + a > 0 \forall x \in \mathbb{R}$

$\Rightarrow D = (18)^2 - 4a < 0$

$\Rightarrow a > 81$

$\Rightarrow a \in (81, \infty)$

11. (c) $\cos^{-1}\left(-\sin \frac{7\pi}{6}\right) = \cos^{-1}\left(-\sin\left(\pi + \frac{\pi}{6}\right)\right)$

$= \cos^{-1}\left(\sin \frac{\pi}{6}\right)$

$= \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

12. (d) $\sin^{-1}\left(-\sin \frac{50\pi}{9}\right) = \sin^{-1}\left(\sin\left(-6\pi + \frac{4\pi}{9}\right)\right)$

$= \sin^{-1}\left(\sin \frac{4\pi}{9}\right)$

$= \frac{4\pi}{9}$

$\cos^{-1}\cos\left(-\frac{31\pi}{9}\right) = \cos^{-1}\cos\left(\frac{31\pi}{9}\right)$

$= \cos^{-1}\cos\left(4\pi - \frac{5\pi}{9}\right) = \cos^{-1}\cos \frac{5\pi}{9} = \frac{5\pi}{9}$

Hence, given equation equals to $\sec\left(\frac{4\pi}{9} + \frac{5\pi}{9}\right) = \sec \pi = -1$.

13. (a) $y = (\sin^{-2}(\sin x))^2 - \sin^{-1}(\sin x)$

$= \left(\sin^{-1}(\sin x) - \frac{1}{2}\right)^2 - \frac{1}{4}$

For maximum value of y , $\sin^{-1}(\sin x) = -\frac{\pi}{2}$

$\Rightarrow y = \left(-\frac{\pi}{2} + \frac{1}{2}\right)^2 - \frac{1}{4} = \frac{\pi}{4} (\pi + 2)$

14. (a) Solution of $y = \sqrt{y}$ is $y = 1$ and $y = 0$

$\Rightarrow \sin^{-1}|\sin x| = 0$ or 1

$\sin^{-1}|\sin x|$ is periodic with period π

In $(0, \pi)$, if $\sin^{-1}|\sin x| = 1$, $x = 1$ or $x = \pi - 1$

$\therefore$ General solution is $x = n\pi \pm 1$, $n \in \mathbb{Z}$

If $\sin^{-1}|\sin x| = 0 \Rightarrow x = n\pi$, $n \in \mathbb{Z}$

15. (c) We have $\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

Let $\sin^{-1}\frac{\sqrt{63}}{8} = \theta$

$\therefore \frac{\sqrt{63}}{8} = \sin \theta$

$\Rightarrow \cos \theta = \frac{1}{8}$

$\therefore \cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}} = \sqrt{\frac{1 + \frac{1}{8}}{2}} = \frac{3}{4}$

$\Rightarrow \sin \frac{\theta}{4} = \sqrt{\frac{1 - \cos \frac{\theta}{2}}{2}} = \sqrt{\frac{1 - \frac{3}{4}}{2}} = \frac{1}{2\sqrt{2}}$

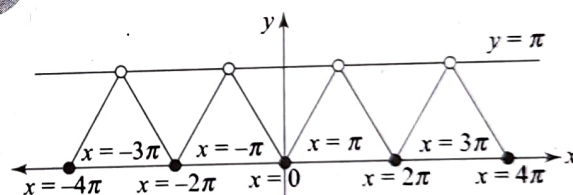
Multiple Correct Answers Type

16. (a, d)

$$f(x) = \cos^{-1}\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)$$

$= \cos^{-1}(\cos x)$, $x \in \mathbb{R} - \{x | x = (2n+1)\pi, n \in \mathbb{Z}\}$

Graph of the functions is as shown in the following figure.



Graph of $f(x)$

From the graph, range of $f(x)$ is $[0, \pi]$

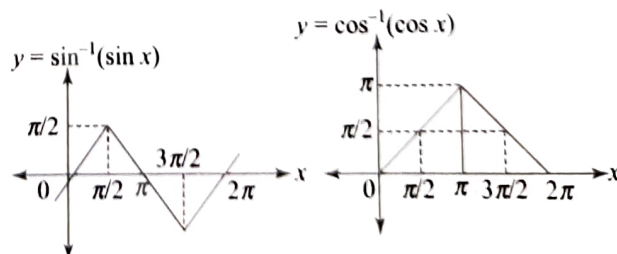
Also, $f(x)$ is periodic with period 2π .

Clearly $f(x) = \pi$ has no real roots.

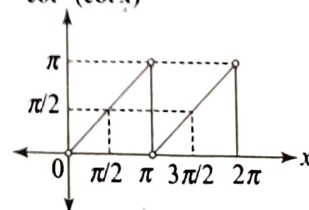
$y = f(x)$ is not identical with $y = \cos^{-1}(\cos x)$

17. (a, c, d)

First draw the graphs of $y = f(x)$, $y = g(x)$ and $y = h(x)$ for $x \in [0, 2\pi]$



$y = \cot^{-1}(\cot x)$



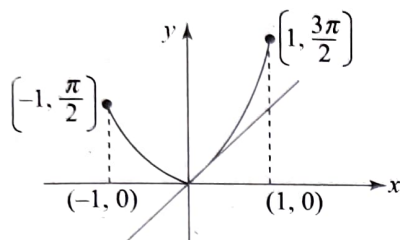
From the graphs, we can see that options (a), (c) and (d) are correct.

Comprehension Type

18. (a), 19. (b), 20. (c)

$$f(x) = \begin{cases} 3 \sin^{-1} x; & x \in [0, 1] \\ -\sin^{-1} x; & x \in [-1, 0) \end{cases}$$

The graph of the function $y = f(x)$ is as shown in the following figure



From the graph $f(x) = x$ has only one solution, $x = 0$

Range of $y = f(x)$ is $[0, 3\pi/2]$

For $f(x) = k$ having two solutions, $k \in (0, \pi/2]$

DPP 5.2

Single Correct Answer Type

1. (d) (a) $\sin \cos^{-1} \tan \cot^{-1} x = \sin \cos^{-1} \frac{1}{x} = \sqrt{1 - \frac{1}{x^2}}$

(b) $\cos \tan^{-1} \cot \sin^{-1} x = \cos \tan^{-1} \frac{\sqrt{1-x^2}}{x} = x$

(c) $\tan \cot^{-1} \sin \cos^{-1} x = \tan \cot^{-1} \sqrt{1-x^2} = \frac{1}{\sqrt{1-x^2}}$

(d) $\cot \sin^{-1} \cos \tan^{-1} x = \cot \sin^{-1} \frac{1}{\sqrt{1+x^2}} = x$

2. (d) Let $\tan^{-1} \frac{x}{2} = \theta$

$$\Rightarrow \tan \theta = \frac{x}{2}$$

$$\Rightarrow \cos \left(\tan^{-1} \frac{x}{2} \right) = \cos \theta = \frac{2}{\sqrt{4+x^2}}$$

$$\Rightarrow f(x) = \tan \left[\sin^{-1} \frac{2}{\sqrt{4+x^2}} \right] = \frac{2}{x}$$

If $x > 0$, then $f(x) = \tan \left(\tan^{-1} \frac{2}{x} \right) = \frac{2}{x}$

If $x < 0$, then $f(x) = \tan \left(\tan^{-1} \left(\frac{-2}{x} \right) \right) = \frac{-2}{x}$

$$\Rightarrow f(x) = \frac{2}{|x|}$$

3. (d) $\sin^{-1} 2x = \pi - \cos^{-1} 3x$
 $\Rightarrow \sin^{-1} 2x = \cos^{-1} (-3x)$

$$\Rightarrow \sqrt{1-4x^2} = -3x \quad (x < 0)$$

$$\Rightarrow x^2 = \frac{1}{13}$$

$$\Rightarrow x = \frac{-1}{\sqrt{13}}$$

But for $x < 0$, $\pi/2 < \cos^{-1} 3x < \pi$ and $-\pi/2 < \sin^{-1} 2x < 0$
 $\therefore$ L.H.S. $\neq \pi$

Hence, equation has no solution.

4. (b) $(\cot^{-1} \alpha)x^2 - (\tan^{-1} \alpha)^{3/2} x + 2(\cot^{-1} \alpha)^2 = 0$

Equation has real roots

$$\therefore D \geq 0 \Rightarrow (\tan^{-1} \alpha)^3 - 8(\cot^{-1} \alpha)^3 \geq 0$$

$$\therefore \tan^{-1} \alpha \geq 2 \cot^{-1} \alpha = \pi - 2 \tan^{-1} \alpha$$

$$\Rightarrow \tan^{-1} \alpha \geq \frac{\pi}{3} \Rightarrow \alpha \geq \sqrt{3}$$

Sum of roots > 0

$$\Rightarrow \frac{(\tan^{-1} \alpha)^{3/2}}{2 \cot^{-1} \alpha} > 0$$

$$\tan^{-1} \alpha > 0 \Rightarrow \alpha > 0$$

Product of roots > 0

$$\Rightarrow 2 \cot^{-1} \alpha > 0 \Rightarrow \alpha \in \mathbb{R}$$

$$\Rightarrow \alpha \geq \sqrt{3}$$

5. (c) $\sin^{-1} x - \frac{1}{\sin^{-1} x} = \cos^{-1} x - \frac{1}{\cos^{-1} x}$

$$\Rightarrow (\sin^{-1} x - \cos^{-1} x) \frac{(\sin^{-1} x \cdot \cos^{-1} x + 1)}{\sin^{-1} x \cdot \cos^{-1} x} = 0$$

$$\Rightarrow \sin^{-1} x = \cos^{-1} x \text{ or } \sin^{-1} x \cos^{-1} x + 1 = 0$$

$$\Rightarrow x = \frac{1}{\sqrt{2}} \text{ or } \sin^{-1} x \left(\frac{\pi}{2} - \sin^{-1} x \right) + 1 = 0$$

$$\Rightarrow x = \frac{1}{\sqrt{2}} \text{ or } \sin^{-1} x = \frac{\frac{\pi}{2} \pm \sqrt{\left(\frac{\pi^2}{4} + 4\right)}}{2}$$

$$\Rightarrow x = \frac{1}{\sqrt{2}} \text{ or } \sin^{-1} x = \frac{\pi}{4} - \sqrt{\left(1 + \frac{\pi^2}{16}\right)}$$

6. (c) $\tan^{-1} x + \sin^{-1} x \geq \frac{\pi}{2}, x \in [-1, 1]$

$$\Rightarrow \tan^{-1} x \geq \frac{\pi}{2} - \sin^{-1} x$$

$$\Rightarrow \tan^{-1} x \geq \cos^{-1} x$$

$$\Rightarrow \tan^{-1} x \geq \tan^{-1} \frac{\sqrt{1-x^2}}{x}$$

$$\Rightarrow x \geq \frac{\sqrt{1-x^2}}{x}$$

$$\Rightarrow x^4 \geq 1 - x^2$$

S.30 Trigonometry

$$\Rightarrow x^4 + x^2 - 1 \geq 0$$

$$\Rightarrow x \in \left[\sqrt{\frac{\sqrt{5}-1}{2}}, 1 \right]$$

$$7. (d) \sin^{-1}(3x - 4x^3) + \cos^{-1}(4x^3 - 3x) = \frac{\pi}{2}$$

$$\Rightarrow 3x - 4x^3 = 4x^3 - 3x$$

$$\Rightarrow 8x^3 - 6x = 0$$

$$\Rightarrow 2x(4x^2 - 3) = 0$$

$$\Rightarrow x = 0, x = \pm \frac{\sqrt{3}}{2}$$

$$8. (c) f(x) = \begin{cases} \sin^{-1} x + \cos^{-1} x; & 0 \leq x \leq 1 \\ \cos^{-1} x - \sin^{-1} x; & -1 \leq x < 0 \end{cases}$$

$$f(x) = \begin{cases} \pi/2, & 0 \leq x \leq 1 \\ \frac{\pi}{2} - 2\sin^{-1} x, & -1 \leq x < 0 \end{cases}$$

For $-1 \leq x \leq 0$

$$-\pi/2 \leq \sin^{-1} x \leq 0$$

$$\therefore -\pi \leq 2\sin^{-1} x \leq 0$$

$$\therefore 0 \leq -2\sin^{-1} x \leq \pi$$

$$\therefore \pi/2 \leq \pi/2 - 2\sin^{-1} x \leq 3\pi/2$$

$$\therefore \text{Max. value} = 3\pi/2$$

$$\text{and Min. value} = \pi/2$$

$$9. (c) f(x) = \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, \forall x \in [-1, 1]$$

$$\text{If } g(x) = |\sin^{-1} x| + \cos^{-1} |x| = \begin{cases} -\sin^{-1} x + \pi - \cos^{-1} x; & -1 \leq x < 0 \\ = \pi - \frac{\pi}{2} = \frac{\pi}{2} \\ \sin^{-1} x + \cos^{-1} x; & 0 \leq x \leq 1 \\ = \frac{\pi}{2} \end{cases}$$

$$10. (d) \sin^{-1}\left(\sqrt{\frac{3x-1}{25}}\right) + \sin^{-1}\left(\sqrt{\frac{3x+1}{25}}\right) = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\left(\sqrt{\frac{3x-1}{25}}\right) = \frac{\pi}{2} - \sin^{-1}\left(\sqrt{\frac{3x+1}{25}}\right) = \cos^{-1}\left(\sqrt{\frac{3x+1}{25}}\right)$$

$$\Rightarrow \sin^{-1}\left(\sqrt{\frac{3x-1}{25}}\right) = \sin^{-1}\left(\sqrt{1 - \frac{3x+1}{25}}\right)$$

$$\Rightarrow \sin^{-1}\left(\sqrt{\frac{3x-1}{25}}\right) = \sin^{-1}\left(\sqrt{\frac{24-3x}{25}}\right)$$

$$\Rightarrow \frac{3x-1}{25} = \frac{24-3x}{25}$$

$$\Rightarrow x = \frac{25}{6}$$

$$11. (b) \sin^{-1} x + \cos^{-1} x + \pi(|x| - 2) = k\pi, x \in [-1, 1]$$

$$\Rightarrow \frac{\pi}{2} + \pi(|x| - 2) = k\pi$$

$$\Rightarrow \frac{1}{2} + |x| - 2 = k$$

$$\Rightarrow |x| = k + \frac{3}{2} \in [0, 1]$$

$$\Rightarrow 0 \leq k + \frac{3}{2} \leq 1$$

$$\Rightarrow -\frac{3}{2} \leq k \leq -\frac{1}{2}$$

$$12. (c) (\sin x + \cos^{-1} x) - (\cos x - \sin^{-1} x) \geq \frac{\pi}{2}$$

$$\therefore \sin x - \cos x + \frac{\pi}{2} \geq \frac{\pi}{2}$$

$$\text{or } \sin x - \cos x \geq 0$$

$$\text{or } \sin x \geq \cos x$$

Clearly domain of inequality is $x \in [-1, 1]$

$$\therefore x \in \left[\frac{\pi}{4}, 1 \right]$$

$$13. (d) f(x) = \sin^{-1} x - \left(\frac{\pi}{2} - \tan^{-1} x \right) + (x+1)^2 + 5$$

$$= \sin^{-1} x + \tan^{-1} x - \frac{\pi}{2} + (x+1)^2 + 5$$

Domain of $f(x)$ is $[-1, 1]$ and $\sin^{-1} x$, $\tan^{-1} x$ and $(x+1)^2$ are all increasing in $[-1, 1]$

$\therefore f(x)$ is increasing

$$f(-1) = \frac{-5\pi}{4} + 5 \text{ and } f(1) = \frac{\pi}{4} + 9$$

$$\therefore \text{Range of } f(x) \text{ is } \left[\frac{-5\pi}{4} + 5, \frac{\pi}{4} + 9 \right]$$

Multiple Correct Answers Type

14. (a, c)

$$\sin^{-1}\left(\frac{\sqrt{x}}{2}\right) + \sin^{-1}\left(\sqrt{1 - \frac{x}{4}}\right) + \tan^{-1} y = \frac{2\pi}{3}, x \in [0, 4]$$

$$\therefore \sin^{-1}\left(\frac{\sqrt{x}}{2}\right) + \cos^{-1}\left(\frac{\sqrt{x}}{2}\right) + \tan^{-1} y = \frac{2\pi}{3}$$

$$\therefore \frac{\pi}{2} + \tan^{-1} y = \frac{2\pi}{3}$$

$$\therefore \tan^{-1} y = \frac{2\pi}{3} - \frac{\pi}{2} = \frac{\pi}{6}$$

$$\Rightarrow y = \frac{1}{\sqrt{3}}$$

$$\therefore \text{maximum value of } (x^2 + y^2) = 16 + \frac{1}{3} = \frac{49}{3}$$

$$\text{and minimum value of } (x^2 + y^2) = (0)^2 + \frac{1}{3} = \frac{1}{3}$$

15. (a, b)

$$\begin{aligned}\cos^{-1}\left(\sqrt{\frac{a-x}{a-b}}\right) &= \sin^{-1}\left(\sqrt{1-\frac{a-x}{a-b}}\right) \\ &= \sin^{-1}\left(\sqrt{\frac{x-b}{a-b}}\right)\end{aligned}$$

$\therefore$ We must have $\frac{a-x}{a-b} > 0$ and $\frac{x-b}{a-b} > 0$

$$0 \leq \frac{a-x}{a-b} < 1 \text{ and } 0 \leq \frac{x-b}{a-b} < 1$$

From both, we get $0 \leq \frac{a-x}{a-b} < 1$

$\therefore a > x > b$ or $a < x < b$

DPP 5.3

Single Correct Answer Type

1. (c) $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\frac{3}{5}$

$$\Rightarrow \tan^{-1}\frac{6}{1+x^2-9} = \tan^{-1}\frac{3}{4}$$

$$\Rightarrow 24 = 3x^2 - 24$$

$$\Rightarrow 3x^2 = 48 \Rightarrow x = \pm 4$$

2. (d) $\tan^{-1}\left(\frac{1}{x}\right) + \tan^{-1}\left(\frac{1}{y}\right) = \frac{1}{7}$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{x} + \frac{1}{y}}{1 - \frac{1}{xy}}\right) = \tan^{-1}\left(\frac{1}{7}\right)$$

$$\Rightarrow \frac{x+y}{xy-1} = \frac{1}{7}$$

$$\Rightarrow 7x+7y=xy-1$$

$$\Rightarrow (7-y)x = -7y-1$$

$$\Rightarrow x = \frac{7y+1}{y-7} = 7 + \frac{50}{y-7}$$

Here, $y = 8, 9, 12, 17, 32, 57$ satisfy above equation.

3. (c) $\sin^{-1}(1-x) + 2\sin^{-1}x = \frac{\pi}{2}$

$$\Rightarrow \sin^{-1}(1-x) = \left(\frac{\pi}{2} - 2\sin^{-1}x\right)$$

$$\Rightarrow 1-x = \sin\left(\frac{\pi}{2} - 2\sin^{-1}x\right)$$

$$\Rightarrow 1-x = \sin\frac{\pi}{2}\cos(2\sin^{-1}x) - \cos\frac{\pi}{2}\sin(2\sin^{-1}x)$$

$$\Rightarrow 1-x = \cos(2\sin^{-1}x)^2$$

$$\Rightarrow 1-x = \cos(\cos^{-1}(1-2x^2)) \Rightarrow 2x^2 - x = 0$$

$$\Rightarrow x = 0 \text{ or } x = 1/2$$

4. (b) $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) = \pi - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \pi - 2\tan^{-1}x$ (as $x > 1$)

$$\tan^{-1}\left(\frac{2x}{x^2-1}\right) = -\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \pi - 2\tan^{-1}x$$
 (as $x > 1$)

$$\therefore \text{Given equation is } 2\pi - 4\tan^{-1}x = 2\pi/3$$

$$\therefore \tan^{-1}x = \pi/3$$

$$\therefore x = \sqrt{3}$$

5. (d) $\sin^{-1}x - \sin^{-1}2x = \pm\frac{\pi}{3}$

$$\Rightarrow \sin^{-1}x - \sin^{-1}\left(\pm\frac{\sqrt{3}}{2}\right) = \sin^{-1}2x$$

$$\Rightarrow \sin^{-1}\left[x\sqrt{1-\frac{3}{4}} - \left(\pm\frac{\sqrt{3}}{2}\sqrt{1-x^2}\right)\right] = \sin^{-1}2x$$

$$\Rightarrow \frac{x}{2} - \left(\pm\frac{\sqrt{3}}{2}\sqrt{1-x^2}\right) = 2x$$

$$\Rightarrow -(\pm\sqrt{3}\sqrt{1-x^2}) = 3x$$

On squaring both sides, we get

$$3(1-x^2) = 9x^2$$

$$\Rightarrow 4x^2 = 1$$

$$\Rightarrow x = \pm\frac{1}{2}$$

6. (b) For $x \in [-1, 0]$, $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = -2\tan^{-1}x$

$$\text{and } \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x$$

$$\therefore p = 0 \forall x \in [-1, 0]$$

7. (c) $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \pi - 2\tan^{-1}x$, for $x \geq 1$

$$\text{and } g(x) = \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$$

$$= \pi - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$= \pi - 2\tan^{-1}x, \text{ for } x \geq 0$$

Now $f(10) - g(100)$

$$= (\pi - 2\tan^{-1}(10)) - (\pi - 2\tan^{-1}(100))$$

$$= 2(\tan^{-1}(100) - \tan^{-1}(10))$$

8. (c) When $x < 0$, we have $\tan^{-1}(6x) = \frac{\pi}{4}$

$$\Rightarrow x = \frac{1}{6}, \text{ which is not possible}$$

$$\text{When } x \geq 0, \text{ we have } \frac{\pi}{4} = \tan^{-1}(6x) - \tan^{-1}(x)$$

$$\Rightarrow \frac{6x-x}{1+6x^2} = 1$$

$$\Rightarrow x = \frac{1}{2}, \frac{1}{3}$$

S.32 Trigonometry

9. (d) $\sin^{-1} \frac{3x}{5} + \sin^{-1} \frac{4x}{5} = \sin^{-1} x$

$$\Rightarrow \sin^{-1} \left(\frac{3x}{5} \sqrt{1 - \frac{16x^2}{25}} + \frac{4x}{5} \sqrt{1 - \frac{9x^2}{25}} \right) = \sin^{-1} x$$

$$\Rightarrow \frac{3x}{5} \frac{\sqrt{25-16x^2}}{5} + \frac{4x}{5} \frac{\sqrt{25-9x^2}}{5} = x$$

$$\Rightarrow x = 0 \text{ or } 3\sqrt{25-16x^2} + 4\sqrt{25-9x^2} = 25$$

$$\text{Now } \sqrt{225-144x^2} + \sqrt{400-144x^2} = 25$$

$$\Rightarrow \frac{175}{\sqrt{400-144x^2} - \sqrt{225-144x^2}} = 25$$

$$\Rightarrow \sqrt{400-144x^2} - \sqrt{225-144x^2} = 7$$

$$\text{From (1) and (2), } \sqrt{400-144x^2} = 16$$

$$\Rightarrow 400 - 144x^2 = 256$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = \pm 1$$

10. (a) For $x \in \left[-1, \frac{-1}{\sqrt{2}}\right]$,

$$y = -\pi - 2 \sin^{-1} x$$

$$\Rightarrow \sin^{-1} x = \frac{-\pi}{2} - \frac{y}{2}$$

$$\Rightarrow f^{-1}(y) = -\sin\left(\frac{\pi}{2} + \frac{y}{2}\right) = -\cos \frac{y}{2}$$

11. (c) $S = \sum_{n=1}^{\infty} \cot^{-1}(n^2 - 3n + 3)$

$$= \sum_{n=1}^{\infty} \tan^{-1} \left(\frac{1}{1+n^2-3n+2} \right)$$

$$= \sum_{n=1}^{\infty} \tan^{-1} \left(\frac{(n-1) - (n-2)}{1+(n-1)(n-2)} \right)$$

$$= \sum_{n=1}^{\infty} (\tan^{-1}(n-1) - \tan^{-1}(n-2))$$

$$= \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$

12. (c) $T_n = \tan^{-1} \left(\frac{2n+2}{1+(n^2+2n)(n^2+2n+1)} \right)$

$$= \tan^{-1} \left(\frac{2n+2}{1+n(n+2)(n+1)(n+1)} \right)$$

$$= (\tan^{-1}(n+1)(n+2) - \tan^{-1} n(n+1))$$

$$\therefore S_n = \sum_{n=1}^n T_n = (\tan^{-1}(n+1)(n+2) - \tan^{-1} 2)$$

$$\text{So, } \lim_{n \rightarrow \infty} S_n = \left(\frac{\pi}{2} - \tan^{-1} 2 \right) = \cot^{-1} 2 = \sin^{-1} \left(\frac{1}{\sqrt{5}} \right)$$

13. (d) We have $2 \sin^{-1} \left(\frac{2x}{1+x^2} \right) - \pi x^3 = 0$

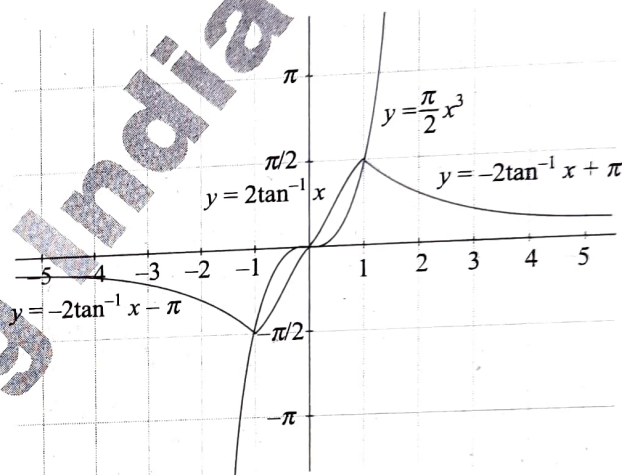
$$\text{or } \sin^{-1} \left(\frac{2x}{1+x^2} \right) = \frac{\pi}{2} x^3$$

Now we know that

$$\sin^{-1} \left(\frac{2x}{1+x^2} \right) = \begin{cases} -2 \tan^{-1} x - \pi, & x < -1 \\ 2 \tan^{-1} x, & -1 \leq x \leq 1 \\ -2 \tan^{-1} x + \pi, & x > 1 \end{cases}$$

To find the number of solutions of the equation, we draw the

graphs of $y = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$ and $y = \frac{\pi}{2} x^3$



Clearly, there are 3 solutions of the equation $\{-1, 0, 1\}$.

Multiple Correct Answers Type

14. (a, b, c)

$$\therefore \sin [2 \cos^{-1} \{ \cot (2 \tan^{-1} x) \}] = 0$$

$$\Rightarrow 2 \cos^{-1} \{ \cot (2 \tan^{-1} x) \} = n\pi, n \in \mathbb{I}$$

$$\Rightarrow \cos^{-1} \{ \cot (2 \tan^{-1} x) \} = \frac{n\pi}{2}$$

$$\Rightarrow \cos^{-1} \{ \cot (2 \tan^{-1} x) \} = 0, \frac{\pi}{2}, \pi$$

$$(\because 0 \leq \cos^{-1} x \leq \pi)$$

$$\Rightarrow \cot \left\{ \tan^{-1} \left(\frac{2x}{1-x^2} \right) \right\} = 1, 0, -1$$

$$\Rightarrow \cot \left\{ \cot^{-1} \left(\frac{1-x^2}{2x} \right) \right\} = 1, 0, -1$$

$$\Rightarrow \frac{1-x^2}{2x} = 1, 0, -1$$

$$\Rightarrow 1-x^2 = 2x, 0, -2x$$

$$\Rightarrow x^2 + 2x - 1 = 0, x^2 - 1 = 0, x^2 - 2x - 1 = 0$$

$$\Rightarrow x = -1 \pm \sqrt{2}, x = \pm 1, x = 1 \pm \sqrt{2}$$

15. (b, c, d)

$$\tan^{-1} \frac{a}{x} + \tan^{-1} \frac{b}{x} + \tan^{-1} \frac{c}{x} + \tan^{-1} \frac{d}{x} = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1} \frac{a}{x} + \tan^{-1} \frac{b}{x} = \frac{\pi}{2} - \left(\tan^{-1} \frac{c}{x} + \tan^{-1} \frac{d}{x} \right)$$

$$\Rightarrow \tan^{-1} \frac{\frac{a}{x} + \frac{b}{x}}{1 - \frac{ab}{x^2}} = \frac{\pi}{2} - \tan^{-1} \frac{\frac{c}{x} + \frac{d}{x}}{1 - \frac{cd}{x^2}}$$

$$\Rightarrow \tan^{-1} \frac{(a+b)x}{x^2 - ab} = \cot^{-1} \frac{(c+d)x}{x^2 - cd}$$

$$\Rightarrow \frac{(a+b)x}{x^2 - ab} = \frac{x^2 - cd}{(c+d)x}$$

$$\Rightarrow x^4 - (ab + ac + ad + bc + bd + cd)x^2 + abcd = 0$$

This equation has roots x_1, x_2, x_3, x_4

$$\therefore \Sigma x_1 = 0, \Sigma x_1 x_2 = -\Sigma ab, \Sigma x_1 x_2 x_3 = 0 \text{ and } x_1 x_2 x_3 x_4 = abcd$$

$$\therefore \Sigma \frac{1}{x_1} = 0$$

$$\text{and } (x_2 + x_3 + x_4)(x_3 + x_4 + x_1)(x_4 + x_1 + x_2)(x_1 + x_2 + x_3) \\ = x_1 x_2 x_3 x_4$$

16. (a, b, c)

$$\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \frac{1}{3} = \frac{\pi}{4} - \tan^{-1} \frac{1}{2} = \frac{\pi}{4} - \cot^{-1} 2$$

$$\text{Also } 2 \tan^{-1} \frac{1}{3} = \tan^{-1} \frac{\frac{2}{3}}{1 - \frac{1}{9}} = \tan^{-1} \frac{3}{4}$$

$$= \sin^{-1} \frac{3}{5} = \frac{\pi}{2} - \cos^{-1} \frac{4}{5}$$

CHAPTER 6

DPP 6.1

Single Correct Answer Type

1. (a) $\frac{c}{\sin 110^\circ} = \frac{b}{\sin 15^\circ} = \frac{a}{\sin 55^\circ} = k$

Now $c^2 - a^2$
 $= k^2 \sin^2 110^\circ - k^2 \sin^2 55^\circ$
 $= k^2 \sin 165^\circ \sin 55^\circ$
 $= (k \sin 55^\circ)(k \sin 15^\circ)$
 $= ab$

2. (b) $2a = \sqrt{3}b + c$

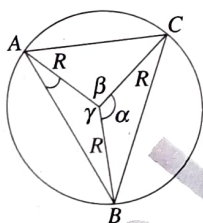
$\Rightarrow a = \frac{\sqrt{3}}{2}b + \frac{1}{2}c$

$\Rightarrow \sin A = \frac{\sqrt{3}}{2} \sin B + \frac{1}{2} \sin C$

$\Rightarrow \cos C = \frac{\sqrt{3}}{2}, \cos B = \frac{1}{2}$

3. (a) We have, $\pi R^2 = 20$ (Given)

Now, $\frac{1}{2} R^2 (\sin \alpha + \sin \beta + \sin \gamma) = 8$ (Given)



$\therefore \sin \alpha + \sin \beta + \sin \gamma = \frac{16}{R^2} = \frac{(16)\pi}{20} = \frac{4\pi}{5}$

4. (a) $bc = 2b^2 \cos A + 2c^2 \cos A - 4bc \cos^2 A$

$\Rightarrow bc = 2 \cos A (b^2 + c^2 - 2bc \cos A)$

$\Rightarrow bc = (2 \cos A) a^2$

$\Rightarrow \frac{bc}{2a^2} = \cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\Rightarrow b^2 c^2 = a^2 (b^2 + c^2 - a^2)$

$\Rightarrow (a^2 - b^2)(a^2 - c^2) = 0$

Thus, triangle is isosceles.

5. (d) $\sin^2 A + \sin^2 C = 1001 \sin^2 B$

$\Rightarrow a^2 + c^2 = 1001 b^2$ (using sine rule)

Now, $\frac{2(\tan A + \tan C) \cdot \tan^2 B}{\tan A + \tan B + \tan C}$
 $= \frac{2(\tan A + \tan C) \cdot \tan^2 B}{\tan A \cdot \tan B \cdot \tan C}$
 $= 2 \left(\frac{\cot A + \cot C}{\cot B} \right)$
 $= \frac{2(\cos A \sin C + \sin A \cos C)}{\sin A \cdot \sin C \cdot \cos B} \sin B$
 $= \frac{2 \sin(\pi - B) \cdot \sin B}{\sin A \sin C \cos B}$
 $= \frac{2 \sin^2 B}{\sin A \sin C \cos B}$
 $= \frac{2 \times 2b^2}{2ac \cdot \cos B}$
 $= \frac{2 \times 2b^2}{a^2 + c^2 - b^2} = \frac{2 \times 2b^2}{1000b^2} = \frac{1}{250}$

6. (a) Let $a > b > c$, given that $b + c > a$ and $b^2 + c^2 > a^2$

$\therefore b^2 + c^2 - a^2 > 0$

$\therefore 2bc \cos A > 0$

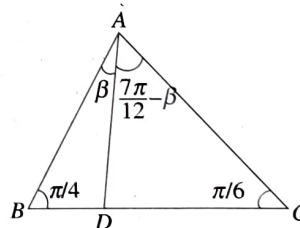
$\therefore \cos A > 0$

$\therefore A$ is acute angle.

Similarly $\cos B > 0, \cos C > 0$

So, $\triangle ABC$ is an acute angled triangle.

7. (b)



In $\triangle ABD$,

$\frac{\sin \beta}{\frac{1}{4} BC} = \frac{\sin \frac{\pi}{4}}{AD}$ and $\frac{\sin \left(\frac{7\pi}{12} - \beta \right)}{\frac{3}{4} BC} = \frac{\sin \frac{\pi}{6}}{AD}$

Dividing, we get

$\frac{\sin \left(\frac{7\pi}{12} - \beta \right)}{\sin \beta} = \frac{3\sqrt{2}}{2}$

$\Rightarrow \frac{\sin \left(\frac{5\pi}{12} + \beta \right)}{\sin \beta} = \frac{3\sqrt{2}}{2}$

$\Rightarrow \sin \left(\frac{5\pi}{12} \right) \cot \beta + \cos \frac{5\pi}{12} = \frac{3\sqrt{2}}{2}$

$\Rightarrow \cot \beta = 4\sqrt{3} - 5$

$$\text{In } \triangle ABC, \frac{AB}{\sin \frac{\pi}{6}} = \frac{AC}{\sin \frac{\pi}{4}}$$

$$\Rightarrow \sqrt{2} AB = AC$$

$$\Rightarrow \left(\sec \frac{\pi}{4}\right) AB = \left(\cot \frac{\pi}{4}\right) AC$$

Multiplying equations (1) and (2), we get

$$\left(\sec \frac{\pi}{4}\right) AB \cot \beta = \cot \left(\frac{\pi}{4}\right) AC (4\sqrt{3} - 5)$$

8. (c) Let the required angle be ' θ '

$$\cos(\pi - \theta) = \frac{a^2 + a^2 - d_1^2}{2a^2}$$

$$\Rightarrow 2a^2 - d_1^2 = -2a^2 \cos \theta$$

$$\Rightarrow d_1^2 = 4a^2 \cos^2 \frac{\theta}{2}$$

$$\Rightarrow d_1 = 2a \cos \frac{\theta}{2}$$

$$\text{Similarly, } d_2^2 = 2a^2(1 - \cos \theta)$$

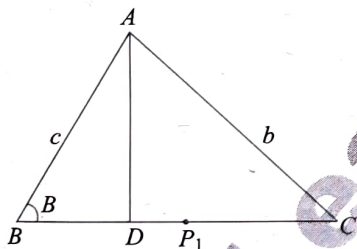
$$\Rightarrow d_2 = 2a \sin \frac{\theta}{2}$$

$$\text{Given; } a^2 = d_1 d_2$$

$$\Rightarrow a^2 = 4a^2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$\Rightarrow \theta = 30^\circ$$

9. (a)



$$P_1D = |BP_1 - BD| = \left| \frac{a}{2} - c \cos B \right|$$

$$= \left| \frac{a}{2} - \frac{c(a^2 + b^2 - c^2)}{2ac} \right|$$

$$= \left| \frac{b^2 - c^2}{2a} \right|$$

$$= \left| \frac{(b-c)(b+c)}{2a} \right|$$

$$\text{Similarly } P_2E = \left| \frac{c^2 - a^2}{2b} \right| = \left| \frac{(c-a)(c+a)}{2b} \right|$$

$$\text{and } P_3F = \left| \frac{(a-b)(a+b)}{2c} \right|$$

$$\text{Given } a : b : c = 3 : 5 : 4$$

$$\therefore a = 3k, b = 5k, c = 4k$$

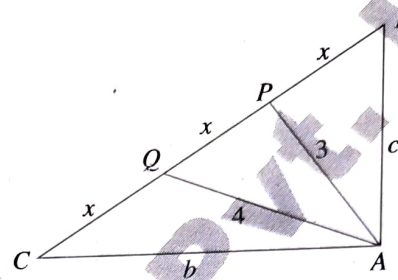
$$P_1D = \frac{3}{2}k, P_2E = \frac{7}{10}k, P_3F = 2k$$

$$\therefore \frac{3}{2}k + 2k + \frac{7k}{10} = \frac{42k}{10} = 42$$

$$\therefore k = 10$$

$$\therefore a + b + c = 120k = 1200$$

(2) 10. (a)



$$\text{Let } BP = PQ = QC = x$$

In $\triangle ABP$, using cosine rule

$$9 = c^2 + x^2 - 2cx \cos B$$

$$\text{But } \cos B = \frac{c}{3x}$$

$$\Rightarrow 9 = x^2 + \frac{c^2}{3}$$

Similarly using cosine rule in $\triangle ACQ$, we get

$$16 = x^2 + \frac{b^2}{3}$$

$$\text{Adding (1) and (2), we get } 25 = 2x^2 + \frac{b^2 + c^2}{3}$$

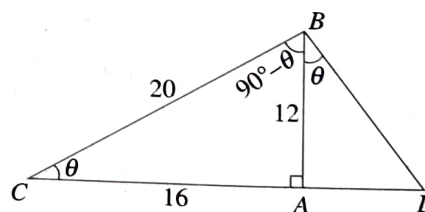
$$\therefore 25 = 2x^2 + \frac{(3x)^2}{3}$$

$$\therefore 25 = 2x^2 + 3x^2$$

$$\therefore x^2 = 5$$

$$\therefore BC = 3x = 3\sqrt{5}$$

11. (a)

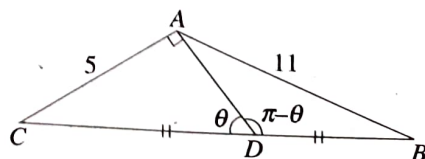


In $\triangle BAD$, $BD = 12 \sec \theta$

$$\therefore \text{From } \triangle BAC, \sec \theta = 5/4$$

$$\therefore BD = 12 \times (5/4) = 15$$

12. (c)



Using sine rule in $\triangle ACD$, we get

$$\frac{5}{\sin \theta} = \frac{CD}{\sin 90^\circ} \text{ or } a = 10 \operatorname{cosec} \theta$$

(1)

Using sine rule in $\triangle ADB$, we get

$$\frac{11}{\sin(\pi - \theta)} = \frac{BD}{\sin(A - 90^\circ)}$$

$$\frac{11}{\sin \theta} = -\frac{a}{2 \cos A}$$

$$\therefore a = -22 \operatorname{cosec} \theta \cos A$$

From (1) and (2), we get

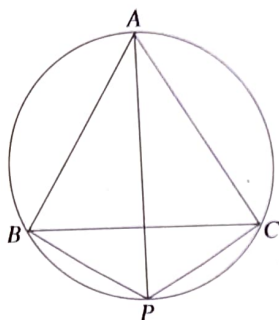
$$\cos A = -5/11$$

$$\text{Also } \cos A = \frac{25 + 121 - a^2}{2(5)(11)} = -\frac{5}{11}$$

$$\therefore a^2 = 196$$

$$\therefore a = 14$$

13. (d) $PA = PB + PC$ and ABC is equilateral
 $\therefore P$ lies on arc BC of circumcircle of $\triangle ABC$



$$\therefore \angle BPC = 120^\circ$$

Using cosine rule in $\triangle BPC$

$$\therefore \cos 120^\circ = \frac{PB^2 + PC^2 - BC^2}{2PB \cdot PC}$$

$$\Rightarrow -PB \cdot PC = PB^2 + PC^2 - a^2$$

$$\Rightarrow 2PB \cdot PC = 2a^2 - 2(PB^2 + PC^2)$$

Again, $PA = PB + PC$

$$\Rightarrow PA^2 = PB^2 + PC^2 + 2PB \cdot PC$$

$$\Rightarrow PA^2 = PB^2 + PC^2 + 2a^2 - 2(PB^2 + PC^2)$$

$$\Rightarrow \frac{PA^2 + PB^2 + PC^2}{a^2} = 2$$

Multiple Correct Answers Type

14. (a, d)

$$\cos(A - C) + \cos B = 1$$

$$\Rightarrow \cos(A - C) - \cos(A + C) = 1$$

$$\Rightarrow 2 \sin A \sin C = 1$$

Now $a = 2c$, so by sine rule $\sin A = 2 \sin C$

From (1) and (2), we get $4 \sin^2 C = 1$

$$\Rightarrow \sin C = \frac{1}{2}$$

$$\Rightarrow C = \left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\}$$

15. (a, b)

$$\sin^2 A + \sin^2 B = \sin^2 C$$

$$\Rightarrow a^2 + b^2 = c^2$$

$$\Rightarrow C = \frac{\pi}{2} \text{ and } A, B < \frac{\pi}{2}$$

$$\text{Since } A + B = \frac{\pi}{2} \therefore \tan A \tan B = 1$$

$$\text{Also } \sin A > \sin^2 A, \sin B > \sin^2 B$$

$$\Rightarrow \sin A + \sin B > \sin^2 A + \sin^2 B = \sin^2 C = 1$$

16. (b, d)

$$\angle C = 2\angle A \text{ and } b = 2a$$

$$\Rightarrow \sin B = 2 \sin A$$

$$\Rightarrow \sin(A + C) = 2 \sin A$$

$$\Rightarrow \sin 3A = 2 \sin A$$

$$\Rightarrow 3 - 4 \sin^2 A = 2$$

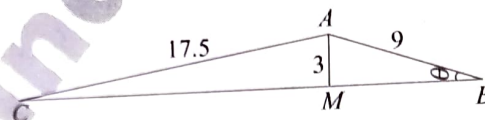
$$\Rightarrow \sin A = \frac{1}{2} \Rightarrow A = 30^\circ$$

$$\therefore C = 60^\circ, B = 90^\circ$$

$$\Rightarrow a^2 + b^2 + c^2 = 8R^2$$

$$\Rightarrow \sin^2 A + \sin^2 B + \sin^2 C = 2$$

17. (a, b, c)



$$\text{From the figure, } \sin \theta = \frac{1}{3}$$

$$\text{In } \triangle ABC, \text{ using sine rule } \frac{17.5}{\sin \theta} = 2R \Rightarrow R = 26.25$$

$$\text{In } \triangle ABM, \text{ using sine rule } \frac{3}{\sin \theta} = 2R_1 \Rightarrow R_1 = 4.5, \text{ where } R_1 \text{ is circumradius of } \triangle ABM.$$

$$\begin{aligned} \text{Also, } \cos A &= \frac{17.5^2 + 9^2 - BC^2}{2(17.5)(9)} \\ &= \frac{17.5^2 + 9^2 - (CM + MB)^2}{2(17.5)(9)} \\ &= \frac{17.5^2 + 9^2 - (17.5^2 - 9) - (9^2 - 9)}{2(17.5)(9)} \\ &= \frac{-2\sqrt{17.5^2 - 9}\sqrt{9^2 - 9}}{2(17.5)(9)} \\ &< 0 \end{aligned}$$

$\therefore A$ is obtuse angle

$\therefore$ orthocenter lies outside $\triangle ABC$.

18. (a, d)

$$\cos \theta = \frac{a - b}{c} = \frac{\sin A - \sin B}{\sin C}$$

$$\begin{aligned} &= \frac{2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}}{2 \sin \frac{C}{2} \cos \frac{C}{2}} \\ &= \frac{\sin \frac{A-B}{2}}{\sin \frac{A+B}{2}} \end{aligned}$$

$$\Rightarrow \sin \theta = \frac{\sqrt{\sin^2 \frac{A+B}{2} - \sin^2 \frac{A-B}{2}}}{\sin \frac{A+B}{2}}$$

$$= \frac{\sqrt{\sin A \sin B}}{\sin \frac{A+B}{2}}$$

$$\Rightarrow \frac{(a+b)\sin \theta}{2\sqrt{ab}} = \frac{\sin A + \sin B}{2\sqrt{\sin A \sin B}} \cdot \frac{\sqrt{\sin A \sin B}}{\sin \frac{A+B}{2}}$$

$$= \frac{2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}}{2 \sin \frac{A+B}{2}}$$

$$= \cos \frac{A-B}{2}$$

$$\frac{c \sin \theta}{2\sqrt{ab}} = \frac{\sin C}{2\sqrt{\sin A \sin B}} \cdot \frac{\sqrt{\sin A \sin B}}{\sin \frac{A+B}{2}}$$

$$= \frac{2 \sin \frac{C}{2} \cos \frac{C}{2}}{2 \sin \frac{A+B}{2}}$$

$$= \cos \frac{A+B}{2}$$

DPP 6.2

Single Correct Answer Type

1. (b) $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} = \frac{2}{3}$

$$\Rightarrow \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} + \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{2}{3}$$

$$\Rightarrow \frac{s-c}{s} + \frac{s-a}{s} = \frac{2}{3}$$

$$\Rightarrow 3(2s-a-c) = 2s$$

$$\Rightarrow 3b = a+b+c$$

$$\Rightarrow a+c = 2b$$

2. (c) $\tan \frac{A}{2} \cdot \tan \frac{B}{2} = \frac{1}{3}$

$$\Rightarrow \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} = \frac{1}{3}$$

$$\Rightarrow \frac{s-c}{s} = \frac{1}{3}$$

$$\Rightarrow 2s = 3c$$

$$\Rightarrow a+b = 2c$$

$$\text{Now } \frac{a+b}{2} \geq \sqrt{ab}$$

$$\Rightarrow c \geq 2$$

3. (d) $\cot \frac{A}{2} \cot \frac{B}{2} = \sqrt{\frac{s(s-a)}{(s-b)(s-c)}} \times \sqrt{\frac{s(s-b)}{(s-c)(s-a)}} = c$

$$\Rightarrow \frac{s}{s-c} = c \Rightarrow \frac{1}{s-c} = \frac{c}{s}$$

$$\text{Similarly } \frac{1}{s-a} = \frac{a}{s}, \quad \frac{1}{s-b} = \frac{b}{s}$$

$$\Rightarrow \frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c} = \frac{a+b+c}{s} = \frac{2s}{s} = 2$$

4. (a)

$$\text{We know } \Delta = sr = (70/2) \times 6 = 210$$

$$\Rightarrow (1/2)ab = 210 \Rightarrow ab = 420$$

$$\text{Now } (a+b)^2 = (70-c)^2$$

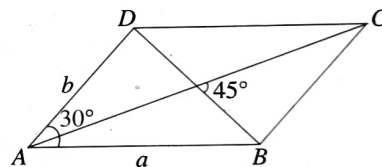
$$\Rightarrow a^2 + b^2 + 2ab = 4900 - 140c + c^2$$

$$\Rightarrow c = \frac{4900 - 2 \times 420}{140} = 29$$

$$[\because a^2 + b^2 = c^2]$$

$$\Rightarrow (a-b)^2 = a^2 + b^2 - 2ab = c^2 - 2ab = 841 - 840 = 1$$

5. (d)



$$\text{Let } AC = d_1 \text{ and } BD = d_2$$

Area of parallelogram is

$$\frac{1}{2} d_1 d_2 \sin 45^\circ = 2 \left(\frac{1}{2} ab \sin 30^\circ \right)$$

$$\Rightarrow d_1 d_2 = \sqrt{2} ab$$

(1)

where

$$d_1^2 = a^2 + b^2 - 2ab \cos 150^\circ = a^2 + b^2 + \sqrt{3} ab$$

$$\text{and } d_2^2 = a^2 + b^2 - 2ab \cos 30^\circ = a^2 + b^2 - \sqrt{3} ab$$

$$\Rightarrow d_1^2 d_2^2 = (a^2 + b^2)^2 - 3a^2 b^2$$

$$\Rightarrow 2a^2 b^2 = (a^2 + b^2)^2 - 3a^2 b^2$$

$$\Rightarrow a^4 + b^4 - 3a^2 b^2 = 0$$

$$\Rightarrow \left(\frac{a}{b} \right)^4 - 3 \left(\frac{a}{b} \right)^2 + 1 = 0$$

$$\Rightarrow \left(\frac{a}{b} \right)^2 = \frac{3 + \sqrt{5}}{2} \quad (\text{as } a > b)$$

$$\Rightarrow \left(\frac{a}{b} \right)^2 = \frac{(\sqrt{5} + 1)^2}{4}$$

$$\Rightarrow \frac{a}{b} = \frac{\sqrt{5} + 1}{2}$$

6. (d) We have

$$2\Delta^2 (a^2 + b^2 + c^2) = a^2 b^2 c^2$$

$$\therefore a^2 + b^2 + c^2 = \left(\frac{abc}{\Delta}\right)^2 \cdot \frac{1}{2} = 8R^2$$

$$\therefore \sin^2 A + \sin^2 B + \sin^2 C = 2$$

$$\therefore \cos 2A + \cos 2B + \cos 2C = -1$$

$$\therefore -1 - 4 \cos A \cos B \cos C = -1$$

$$\therefore \cos A \cos B \cos C = 0$$

$$\therefore \cos A = 0 \text{ or } \cos B = 0 \text{ or } \cos C = 0$$

$\therefore$ Triangle is right angled.

$$7. (d) \Delta \leq \frac{b^2 + c^2}{\lambda}$$

$$\Rightarrow \frac{1}{2} bc \sin A \leq \frac{b^2 + c^2}{\lambda}$$

$$\Rightarrow bc \left(\frac{1}{2} \lambda \sin A - 2 \right) \leq (b - c)^2$$

Since $\sin A \leq 1$, the above inequality will always be satisfied if $\lambda = 4$

$$8. (b) 2\Delta a - b^2 c = c^3$$

$$\Rightarrow 2\Delta a^2 b = abc(b^2 + c^2)$$

$$\Rightarrow \frac{(a^2 b) abc}{2R} = abc(b^2 + c^2)$$

$$\Rightarrow \frac{a^2 b}{2R} = b^2 + c^2$$

$$\Rightarrow a^2 \sin B = b^2 + c^2$$

If $\sin B = 1$, then $a^2 = b^2 + c^2$, which is not possible

$$\therefore \sin B \neq 1$$

$$\begin{aligned} \therefore \cos A &= \frac{b^2 + c^2 - a^2}{2bc} \\ &= \frac{a^2 \sin B - a^2}{2bc} \\ &< 0 \end{aligned}$$

$\therefore A$ is obtuse

$$9. (c) \text{ L.H.S.} = \frac{1}{2} (a^2 (b + c - a) + b^2 (c + a - b) + c^2 (a + b - c))$$

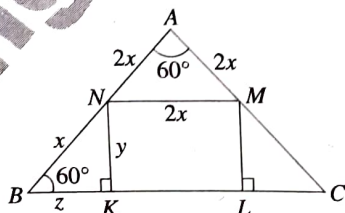
$$= \frac{1}{2} (a(b^2 + c^2 - a^2) + b(c^2 + a^2 - b^2) + c(a^2 + b^2 - c^2))$$

$$= \frac{1}{2} (2abc \cos A + 2abc \cos B + 2abc \cos C)$$

$$= abc \left(1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \right)$$

$$= 4R\Delta \left(1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} \right)$$

10. (b)



$$y = \frac{\sqrt{3}x}{2}, z = \frac{x}{2}$$

Area of ΔBKN is 6

$$\therefore \frac{1}{2} yz = 6 \Rightarrow x^2 = \frac{48}{\sqrt{3}}$$

$$\text{Area of } \Delta ABC = 6 + 6 + 2xy + \frac{1}{2} (2x)(2x) \sin 60^\circ$$

$$= 12 + 2x \frac{\sqrt{3}x}{2} + 2x^2 \frac{\sqrt{3}}{2}$$

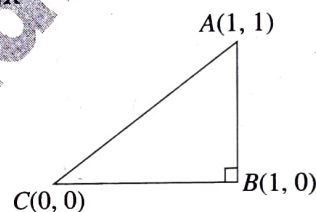
$$= 12 + 2\sqrt{3} \times \frac{48}{\sqrt{3}} = 108$$

Multiple Correct Answers Type

11. (a, b, d)

Area of ΔABC

$$\Delta = \frac{abc}{4R}$$



Since all points are integral, the least area is $\frac{1}{2}$ sq. units as shown in above figure.

$$\therefore \Delta \geq \frac{1}{2}$$

$$\therefore 2R \leq abc$$

12. (a, c)

$$\tan \frac{A+B}{2} = \frac{\frac{5}{6} + \frac{20}{37}}{1 - \frac{5}{6} \cdot \frac{20}{37}} = \frac{305}{122}$$

$$\Rightarrow \tan \frac{C}{2} = \frac{122}{305}$$

$$\Rightarrow \tan \frac{B}{2} > \tan \frac{C}{2}$$

$$\Rightarrow \angle B > \angle C$$

Also $\angle A > \angle B > \angle C$

Hence, $a > b > c$

13. (a, b)

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$= (a - b)^2 + 2ab(1 - \cos C)$$

$$= (a - b)^2 + \frac{4\Delta}{\sin C} (1 - \cos C)$$

Hence, for c to be minimum, $a = b$

$$\text{Also } \Delta = \frac{1}{2} ab \sin C \Rightarrow a^2 = \frac{2\Delta}{\sin C} = b^2$$

DPP 6.3

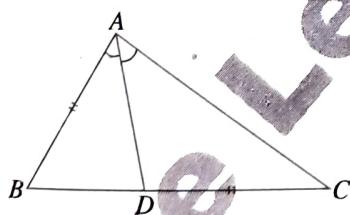
Single Correct Answer Type

1. (c) Given $OA = HA$
 $\Rightarrow R = 2R \cos A$
 $\Rightarrow \cos A = \frac{1}{2}$
 $\Rightarrow A = \frac{\pi}{3}$

2. (c) $r = (s - a) \tan \left(\frac{A}{2} \right)$
 $\therefore r = \left(\frac{AB + BC + CA}{2} - BC \right) \frac{1}{\sqrt{3}}$
 $\therefore r = \frac{1}{\sqrt{3}} \frac{AB - BC + CA}{2}$
 $\therefore 2\sqrt{3} = \frac{AB - BC + CA}{r}$
 $\therefore \left(\frac{AB - BC + CA}{r} \right)^2 = 12$

3. (c) $\Delta = 2R^2 \sin A \sin B \sin C$
 Similarly, $p = 2 \left(\frac{R}{2} \right)^2 \sin 2A \sin 2B \sin 2C$
 $= 4R^2 \sin A \sin B \sin C \cos C \cos B \cos C$
 Now $\frac{p}{\Delta} = 2 \cos A \cos B \cos C \geq \frac{1}{4}$
 where $\cos^2 B + \sin^2 B = 1 \Rightarrow \frac{p}{\Delta} = \frac{1}{4}$
 $\Rightarrow \Delta ABC$ is equilateral
 $\Rightarrow 8(\cos^2 A \cos B + \cos^2 C) = 3$

4. (d)



Since $\angle B = 2\angle C$
 $\sin B = \sin 2C$
 $\Rightarrow \sin B = 2 \sin C \cos C$
 $\Rightarrow b(2ab) = 2c(a^2 + b^2 - c^2)$
 $\Rightarrow ab^2 = ca^2 + cb^2 - c^3$
 $\Rightarrow ab^2 - cb^2 = ca^2 - c^3$
 $\Rightarrow b^2(a - c) = c(a^2 - c^2)$
 $\Rightarrow b^2 = c(a + c)$
 Now $CD = c$ and $BD = a - c$
 Also D is angle bisector
 $\therefore \frac{a - c}{c} = \frac{c}{b}$
 $\Rightarrow c^2 = ab - bc$

$\Rightarrow b^2 - ac = ab - bc$ (using $b^2 = c(a + c)$)
 $\Rightarrow b^2 + bc = ac + ab$
 $\Rightarrow b(b + c) = a(b + c)$
 $\Rightarrow a = b$
 $\Rightarrow \angle A = 2\angle C$

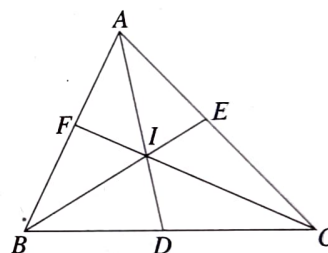
Now $\angle A + \angle B + \angle C = \pi$
 $\therefore 2\angle C + 2\angle C + \angle C = \pi$
 $\therefore \angle C = \frac{\pi}{5} = \frac{180^\circ}{5} = 36^\circ$
 $\therefore \angle A = 72^\circ$

5. (a) Given $R : r = 3 : 1.5 = 2$
 $\Rightarrow \Delta ABC$ must be equilateral.

So $a = b = c = 2R \sin \frac{\pi}{3} = R\sqrt{3}$ (By sine rule)

Now $a \cot^2 A + b^2 \cot^3 B + c^3 \cot^4 C$
 $= R\sqrt{3} \left(\frac{1}{\sqrt{3}} \right)^2 + (R\sqrt{3})^2 \left(\frac{1}{\sqrt{3}} \right)^3 + (R\sqrt{3})^3 \left(\frac{1}{\sqrt{3}} \right)^4$
 $= \frac{R}{\sqrt{3}} + \frac{R^2}{\sqrt{3}} + \frac{R^3}{\sqrt{3}} = \frac{3 + 3^2 + 3^3}{\sqrt{3}} = \frac{39}{\sqrt{3}} = 13\sqrt{3}$

6. (b)



$\frac{ID}{AD} = \frac{ID}{\frac{2bc}{b+c} \cos \frac{A}{2}} = \frac{a(b+c) \sec \frac{A}{2} \cdot ID}{2abc}$

Now $\frac{AI}{ID} = \frac{AB}{BD} = \frac{c}{\frac{ac}{b+c}} = \frac{b+c}{a}$

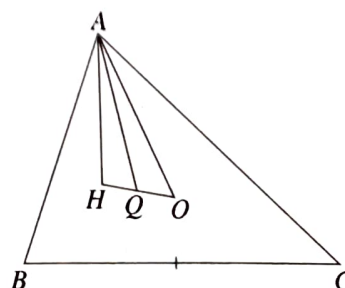
$\therefore \frac{ID}{AD} = \frac{ID}{AI + ID} = \frac{a}{a + b + c}$

$\therefore \frac{ID}{AD} + \frac{IE}{BE} + \frac{IF}{CF} = \frac{a + b + c}{a + b + c} = 1$

$\therefore a(b+c) \sec \frac{A}{2} ID + b(a+c) \sec \frac{B}{2} IE + c(a+b) \sec \frac{C}{2} IF$
 $= 2abc$

$\therefore k = 2$

7. (a)



We know that $OA = R$, $HA = 2R \cos A$ and

$$OH = R\sqrt{1 - 8 \cos A \cos B \cos C}$$

Applying Apollonius theorem to $\triangle AOH$, we get

$$2(AQ)^2 + 2(OQ)^2 = OA^2 + (HA)^2$$

$$\Rightarrow 2(AQ)^2 = R^2 + 4R^2 \cos^2 A - \frac{R^2}{2}(1 - 8 \cos A \cos B \cos C)$$

$$\Rightarrow 4AQ^2 = 2R^2 + 8R^2 \cos^2 A - R^2 + 8R^2 \cos A \cos B \cos C$$

$$\Rightarrow 4AQ^2 = R^2(1 + 8 \cos^2 A + 8 \cos A \cos B \cos C)$$

$$= R^2(1 + 8 \cos A(\cos A + \cos B \cos C))$$

$$= R^2(1 + 8 \cos A(\cos B \cos C - \cos(B + C)))$$

$$= R^2(1 + 8 \cos A \sin B \sin C)$$

8. (a) Distance of O from AC = Distance of I from AC

$$\Rightarrow R \cos B = r$$

$$\Rightarrow \frac{r}{R} = \cos B$$

$$\Rightarrow 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} = \cos B$$

$$\Rightarrow \cos A + \cos B + \cos C - 1 = \cos B$$

$$\Rightarrow \cos A + \cos C = 1$$

$OI = |AE - AD|$ (where E and D are feet of perpendiculars from O and I respectively on AC)

$$= |(b/2) - (s - a)|$$

$$= \frac{|a - c|}{2}$$

$$= R|\sin A - \sin C|$$

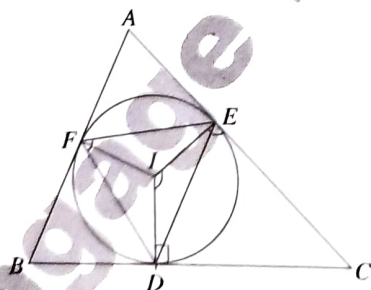
$$= 2R \left| \sin \frac{A - C}{2} \cos \frac{A + C}{2} \right|$$

$$= R \left| \tan \frac{A - C}{2} \cdot 2 \cos \frac{A + C}{2} \cos \frac{A - C}{2} \right|$$

$$= R \left| \tan \frac{A - C}{2} (\cos A + \cos C) \right|$$

$$= R \left| \tan \left(\frac{A - C}{2} \right) \right|$$

9. (d)



Clearly points C, D, I and E are concyclic

$$\therefore \angle EID = \pi - C$$

$$\therefore \text{In } \triangle DEF, \angle DFE = \frac{\pi - C}{2}$$

$$\text{Similarly } \angle EDF = \frac{\pi - A}{2} \text{ and } \angle FED = \frac{\pi - B}{2}$$

$$\begin{aligned} \Delta' &= 2r^2 \sin\left(\frac{\pi - A}{2}\right) \sin\left(\frac{\pi - B}{2}\right) \sin\left(\frac{\pi - C}{2}\right) \\ &= 2r^2 \cos\left(\frac{A}{2}\right) \cos\left(\frac{B}{2}\right) \cos\left(\frac{C}{2}\right) \end{aligned}$$

$$\therefore abc(a + b + c) \frac{\Delta'}{\Delta^3}$$

$$= \frac{abc(a + b + c)}{\Delta^3} 2r^2 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$$

$$= \frac{abc(2s)}{\Delta^3} 2r^2 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$$

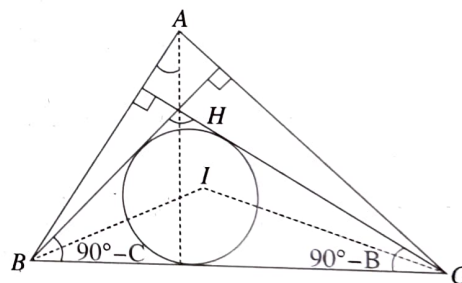
$$= \frac{abc(2s)}{\Delta^3} 2r^2 \sqrt{\frac{s(s-a)}{bc}} \sqrt{\frac{s(s-b)}{ab}} \sqrt{\frac{s(s-c)}{ac}}$$

$$= \frac{4abc}{\Delta^3} r^2 \frac{s}{abc} \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \frac{4s^2}{\Delta^2} r^2$$

$$= 4$$

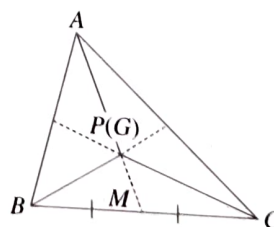
$$\begin{aligned} 10. (b) \angle BIC &= 180^\circ - \frac{90^\circ - C}{2} - \frac{90^\circ - B}{2} \\ &= \frac{B + C}{2} + 90^\circ \end{aligned}$$



Multiple Correct Answers Type

11. (a, b, c, d)

(a)

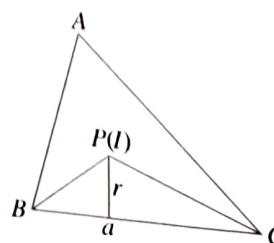


Using properties of median, we have

$$\triangle PBC = \triangle PCA = \triangle PAB$$

$$\therefore \triangle PBC : \triangle PCA : \triangle PAB = 1 : 1 : 1$$

(b)

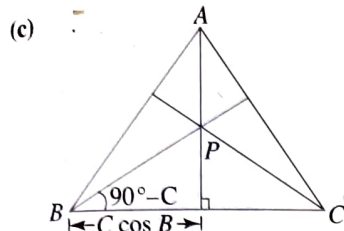


$$\Delta PBC : \Delta PCA : \Delta PAB$$

$$= \frac{1}{2}ar : \frac{1}{2}br : \frac{1}{2}cr$$

$$= a : b : c$$

$$= \sin 45^\circ : \sin 60^\circ : \sin 75^\circ = 2 : \sqrt{6} : (\sqrt{3} + 1)$$



$$\Delta PBC : \Delta PCA : \Delta PAB$$

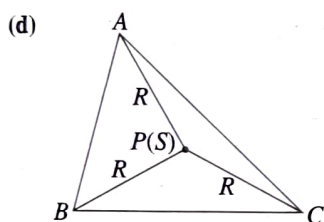
$$= \frac{1}{2}a(2R \cos B \cos C) : \frac{1}{2}b(2R \cos C \cos A)$$

$$: \frac{1}{2}c(2R \cos A \cos B)$$

$$= \sin A \cos B \cos C : \sin B \cos C \cos A : \sin C \cos A \cos B$$

$$= \tan 45^\circ : \tan 60^\circ : \tan 75^\circ$$

$$= 1 : \sqrt{3} : (2 + \sqrt{3})$$



$$\Delta PBC : \Delta PCA : \Delta PAB$$

$$= \frac{1}{2}R^2 \sin 2A : \frac{1}{2}R^2 \sin 2B : \frac{1}{2}R^2 \sin 2C$$

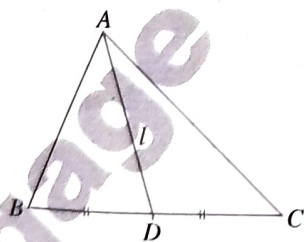
$$= \sin 2A : \sin 2B : \sin 2C$$

$$= \sin 90^\circ : \sin 120^\circ : \sin 150^\circ$$

$$= 2 : \sqrt{3} : 1$$

12. (a, b, c, d)

D is the midpoint of BC



Using Apollonius theorem, we have

$$AB^2 + AC^2 = 2(AD^2 + BD^2)$$

$$\Rightarrow c^2 + b^2 = 2 \left[l^2 + \left(\frac{a}{2} \right)^2 \right] \quad (\text{where } AD = l)$$

$$\Rightarrow 4l^2 = 2b^2 + 2c^2 - a^2$$

$$= b^2 + c^2 + (b^2 + c^2 - a^2)$$

$$= b^2 + c^2 + 2bc \cos A$$

$$= (b^2 + c^2 - a^2) + a^2 + 2bc \cos A$$

$$= 2bc \cos A + a^2 + 2bc \cos A$$

$$= 4bc \cos A + a^2$$

$$\text{Also } 4l^2 = b^2 + c^2 + 2bc \cos A$$

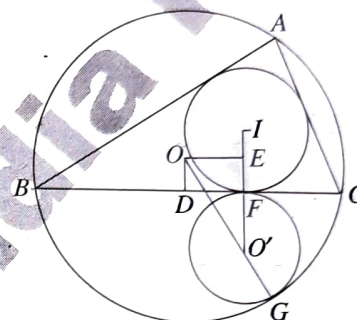
$$= b^2 + c^2 + 2bc \left(1 - 2 \sin^2 \frac{A}{2} \right)$$

$$= (b + c)^2 - 4bc \sin^2 \frac{A}{2}$$

$$= (2s - a)^2 - 4bc \sin^2 \frac{A}{2}$$

13. (a, b, c, d)

From figure, $OO' = OG - O'G = R - r'$



$$OE = DF = |BF - BD| = \left| s - b - \frac{a}{2} \right| = \frac{|c - b|}{2}$$

$$OD = EF = R \cos A$$

From $\triangle OEO'$, using Pythagoras theorem, we get

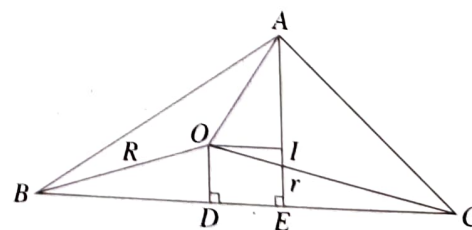
$$(R - r')^2 = (R \cos A + r')^2 + \left(\frac{c - b}{2} \right)^2$$

$$\Rightarrow r' = \frac{bc}{4R} \tan^2 \frac{A}{2}$$

$$= \frac{\Delta}{a} \tan^2 \frac{A}{2}$$

Comprehension Type

14. (b)



$\therefore$ The line joining O and I is parallel to BC

$\therefore OI = DE$ and $OD = IE$

$OD = R \cos A$, $IE = r$

$\therefore r = R \cos A$

$\therefore \cos A = \frac{r}{R} \leq \frac{1}{2}$

$\Rightarrow 0 < \cos A \leq \frac{1}{2} \Rightarrow \frac{\pi}{3} \leq A < \frac{\pi}{2}$

15. (c)
- $ODEI$
- is a square; hence,
- $OD = OI$

$$\text{Now, } OI = \sqrt{R^2 - 2Rr}$$

$$\therefore \sqrt{R^2 - 2Rr} = R \cos A$$

$$\Rightarrow R^2 - 2Rr = R^2 \cos^2 A$$

$$\text{or } 1 - \cos^2 A = \frac{2r}{R} \quad \text{Also } \cos A = \frac{r}{R}$$

$$\Rightarrow 1 - \left(\frac{r}{R}\right)^2 = \frac{2r}{R}$$

$$\Rightarrow \left(\frac{r}{R}\right)^2 + \frac{2r}{R} - 1 = 0$$

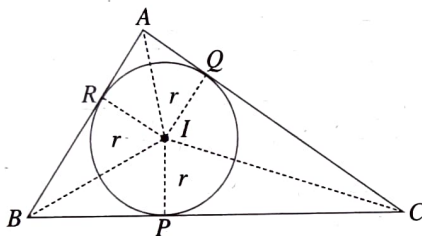
$$\Rightarrow \frac{r}{R} = \sqrt{2} - 1$$

$$16. (d) \therefore \frac{r}{R} = \cos A = \frac{1}{2}$$

$$\therefore A = 60^\circ \text{ but } \frac{r}{R} \leq \frac{1}{2} \text{ in any } \triangle ABC$$

Hence, $\triangle$ is equilateral

17. (b), 18. (c), 19. (d)



$$\text{Let } \tan \frac{A}{2} = x, \tan \frac{B}{2} = y, \tan \frac{C}{2} = z$$

$$\therefore x = \frac{r}{AR}, y = \frac{r}{BP}, z = \frac{r}{CQ}$$

Put the values of AR , BP and CQ in given relation, we get

$$\frac{2x}{r} + \frac{5y}{r} + \frac{5z}{r} = \frac{6}{r}$$

$$\Rightarrow 2x + 5y + 5z = 6 \quad (1)$$

$$\text{Also in the triangle, } xy + yz + zx = 1 \quad (2)$$

If we interchange y and z , then both equations (1) and (2) remain unchanged

$$\Rightarrow \triangle ABC \text{ is isosceles with } \angle B = \angle C \Rightarrow y = z$$

So from (1) and (2), we get

$$x = 3 - 5y \quad (3)$$

$$\text{and } 2xy + y^2 = 1 \quad (4)$$

$$\text{Solving we get, } x = \frac{4}{3}, y = z = \frac{1}{3}$$

$$\therefore AR = \frac{3r}{4}, BP = 3r, CQ = 3r$$

$$\text{Now perimeter } 2s = 2 \cdot AR + 2 \cdot BP + 2 \cdot CQ = \frac{27r}{2}$$

Given perimeter is smallest integer $\Rightarrow r = 2$

$$\therefore s = \frac{27}{2}$$

$$\therefore \Delta = rs = 2 \times \frac{27}{2} = 27 \text{ sq. unit}$$

DPP 6.4

Single Correct Answer Type

1. (b) We know that in a
- $\triangle ABC$

$$r_1 + r_2 + r_3 = 4R + r$$

$$\Rightarrow 4R + r = 9r$$

$$\Rightarrow R = 2r$$

Thus, $\triangle ABC$ is equilateral.

2. (d)
- $r_1 - r = 4R \sin^2 \frac{A}{2}$

$$\text{Now } r_1 - r = 7r - r = 6r = R$$

$$\Rightarrow R = 4R \sin^2 \frac{A}{2}$$

$$\Rightarrow A = \frac{\pi}{3}$$

3. (a)
- $r_2 + r_3 = 4R \cos^2 \frac{A}{2}$

$$\frac{rr_1}{r_2 r_3} = \tan^2 \frac{A}{2}$$

$$\Rightarrow (r_2 + r_3) \sqrt{\frac{rr_1}{r_2 r_3}}$$

$$= 4R \cos^2 \frac{A}{2} \times \tan \frac{A}{2}$$

$$= 2R \sin A$$

$$= a$$

4. (b)
- $r_2 + r_3 = 4R \cos^2 \frac{A}{2}$

$$\angle A = 90^\circ$$

$$\therefore \cos^{-1} \left(\frac{R}{r_2 + r_3} \right)$$

$$= \cos^{-1} \left(\frac{R}{4R \cos^2 \frac{A}{2}} \right)$$

$$= \cos^{-1} \frac{1}{2}$$

$$= 60^\circ$$

5. (b)
- $r_1 = 2r_2 = 3r_3$

$$\Rightarrow \frac{\Delta}{s-a} = \frac{2\Delta}{s-b} = \frac{3\Delta}{s-c} = \frac{\Delta}{k} \text{ (say)}$$

$$\Rightarrow s-a = k, s-b = 2k, s-c = 3k$$

$$\Rightarrow 3s - (a+b+c) = 6k \Rightarrow s = 6k$$

$$\Rightarrow \frac{a}{5} = \frac{b}{4} = \frac{c}{3} = k$$

$$\text{So, } a^2 = b^2 + c^2$$

 $\triangle ABC$ is right angled $\triangle$, $\angle A = 90^\circ$ and D is midpoint of BC ,

$$AD = DC \text{ (radius of circumcircle)}$$

$$\Rightarrow \angle DAC = C \Rightarrow \angle ADC = 180^\circ - 2C$$

$$\Rightarrow \cos \angle ADC$$

$$= \cos (180^\circ - 2C)$$

$$= -\cos 2C$$

$$= -(2 \cos^2 C - 1)$$

$$= 1 - 2 \cos^2 C$$

$$= 1 - 2 \times \frac{16}{25} = \frac{-7}{25}$$

$$\left[\text{from } \triangle ABC, \cos C = \frac{b}{a} = \frac{4}{5} \right]$$

6. (a) $a^2 = b^2 + c^2 - 2bc \cos A$
 or $c^2 - (2b \cos A)c + b^2 - a^2 = 0$
 Above equation has two roots c_1 and c_2
 $\therefore c_1 + c_2 = 2b \cos A$ and $c_1 c_2 = b^2 - a^2$

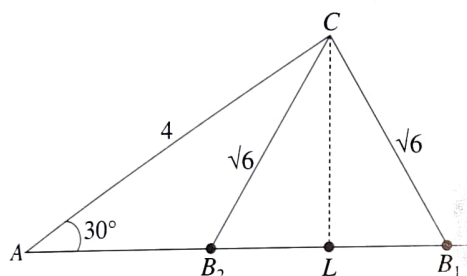
$$\sin B_1 = \sin B_2 = \frac{b \sin A}{a}$$

$$\sin C_1 = \frac{c_1 \sin A}{a}$$

$$\sin C_2 = \frac{c_2 \sin A}{a}$$

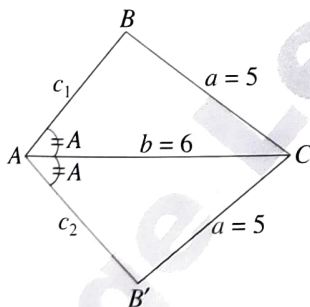
$$\therefore \frac{\sin C_1}{\sin B_1} + \frac{\sin C_2}{\sin B_2} = \frac{c_1 + c_2}{b} = 2 \cos A$$

7. (c) $CL = AC \sin 30^\circ = 2 < \sqrt{6}$
 $\therefore$ Two triangles will be formed



8. (d) $\frac{abc}{a+b+c} = \frac{4R\Delta}{2s} = 2R \cdot r \leq R^2$
 $\therefore$ Maximum value of $\frac{abc}{a+b+c}$ is 4

9. (b)



$$\text{We have } \cos A = \frac{6^2 + c^2 - 5^2}{2 \cdot 6 \cdot c}$$

$$\Rightarrow c^2 - (12 \cos A)c + 11 = 0$$

$$\Rightarrow c_1 + c_2 = 12 \cos A; c_1 c_2 = 11$$

$$\Rightarrow A_1 = \frac{1}{2} bc_1 \sin A; A_2 = \frac{1}{2} bc_2 \sin A$$

$$\text{Given expression} = \frac{(A_1 + A_2)^2 - 2A_1 \cdot A_2 (2 \cos^2 A)}{(A_1 + A_2)^2}$$

$$= 1 - 4 \cos^2 A \left(\frac{A_1 A_2}{(A_1 + A_2)^2} \right)$$

$$= 1 - 4 \cos^2 A \left(\frac{\frac{1}{4} b^2 \sin^2 A c_1 c_2}{\frac{1}{4} b^2 \sin^2 A (c_1 + c_2)^2} \right)$$

$$= 1 - 4 \cos^2 A \left(\frac{11}{144 \times \cos^2 A} \right) = \frac{25}{36}$$

Multiple Correct Answers Type

10. (a, c)

$$r_1 + r_2 = 3R$$

$$\Rightarrow \left(\frac{\Delta}{s-a} \right) + \left(\frac{\Delta}{s-b} \right) = 3 \left(\frac{abc}{4\Delta} \right)$$

$$\Rightarrow \frac{\Delta^2}{(s-a)(s-b)} = \frac{3ab}{4}$$

$$\Rightarrow 4s(s-c) = 3ab$$

$$\Rightarrow \cos \frac{C}{2} = \frac{\sqrt{3}}{2} \Rightarrow \angle C = 60^\circ$$

$$\text{Also, } r_2 + r_3 = 2R$$

$$\Rightarrow \left(\frac{\Delta}{s-b} \right) + \left(\frac{\Delta}{s-c} \right) = 2 \left(\frac{abc}{4\Delta} \right)$$

$$\Rightarrow 2s(s-a) = bc$$

$$\Rightarrow \cos \frac{A}{2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \angle A = 90^\circ \text{ and } \angle B = 30^\circ$$

11. (a, c, d)

As r_1, r_2, r_3 are in H.P.

$$\Rightarrow \frac{s-a}{\Delta}, \frac{s-b}{\Delta}, \frac{s-c}{\Delta} \text{ are in A.P.}$$

$$\Rightarrow s-a, s-b, s-c \text{ are in A.P.}$$

$$\Rightarrow a, b, c \text{ are in A.P.}$$

$$\text{Also, } a+b+c = 24 \text{ (Given)}$$

$$\Rightarrow b = 8, s = 12, c = 16 - a$$

$$\text{Also, } \sqrt{s(s-a)(s-b)(s-c)} = 24 \text{ (given)}$$

$$\Rightarrow 12 \cdot (12-a)(12-8) \cdot (12-(16-a)) = 576$$

$$\Rightarrow (12-a)(a-4) = 12$$

$$\Rightarrow (a-6)(a-10) = 0$$

$$\Rightarrow a = 6 \text{ or } 10$$

Hence, the sides of triangle are 6, 8, 10.

Clearly, the length of longest side of triangle is 10.